A Numerical Investigation of
Sulfate Production and Deposition in
Midlatitude Continental Cumulus Clouds

by

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Program Authorised
to Offer Degree______________________________________

Date_______________________________________
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Hwilm is þurhræse. þæt me on bæce ridere, wun wægforu, wile to þringe laguстрreama full, ···

At times I rush through the dark water vessels that ride upon my back, scattering far the sea-stream's cup,

from the Exeter Book Old English
riddle no. 1, Storm
University of Washington

Abstract

A Numerical Investigation of
Sulfate Production and Deposition in
Midlatitude Continental Cumulus Clouds

by Gregory R. Taylor

Chairperson of the Supervisory Committee: Prof. Marcia B. Baker
Geophysics Program

I have developed a 1.5 dimensional Eulerian cumulus cloud model which incorporates: (a) two in-cloud regions and the environment, (b) a bulk water parameterization including the ice phase, (c) an entrainment formulation based on the turbulent kinetic energy, (d) a sustained sub-cloud forcing, and (e) a parameterization of the scavenging and production of sulfate. This model formulation was chosen in order to provide a simple dynamic framework which permits simultaneous up- and downdrafts in-cloud. It is used to examine the sensitivity of sulfate oxidation and deposition to the physical processes important in the development of midlatitude continental cumulus clouds.

The model is compared against aircraft and radar data for two clouds observed during the Cooperative Convective Precipitation Experiment held in South-central Montana during the summer of 1981. The overall dynamic, thermodynamic and microphysical agreement is good.

The cloud model is then used to examine characteristics of the sulfate cycle. Specifically, I have studied (a) the relative importance of aerosol scavenging to in-cloud production of sulfate through oxidation of sulfur dioxide by ozone and hydrogen...
peroxide, (b) the importance of the entrainment of environmental air in the formation and subsequent deposition of sulfate, (c) the role of ice phase microphysical processes in determining the chemical properties of precipitation, and (d) the chemical effects on clouds due to modification of the environment by previous clouds.

The results indicate for a continental background or moderately polluted atmosphere that aerosol scavenging accounts for between 50 and 80% of the in-cloud sulfate ultimately deposited; that ozone oxidation of sulfur dioxide may be important in the upper regions of the cloud; that although entrainment of environmental air is important in the cloud dynamics and microphysics, it is relatively unimportant in the chemistry, and that neglect of the ice phase when considering chemistry in these clouds may lead to overestimates of about 200% in sulfate deposition.

CHAPTER 1

INTRODUCTION

A great deal of effort is currently directed toward understanding the cloud chemical processes and their effects upon mesoscale wet deposition. Since the lifetimes and spatial dimensions of individual clouds cannot, in general, be explicitly resolved on a mesoscale grid, the cloud processes must be parameterized. Cloud models used in cumulus parameterizations tend to be simple steady state Lagrangian adiabatic parcel models with little or no microphysics (e.g., Kuo, 1974, or Kuo and Anthes, 1984). The cloud models incorporated into the regional deposition models are generally of the same sophistication (see, for example, NCAR/TN-256+STR, 1985) For the most part, these simple models are used because of limitations in computer resources. This constraint will continue to impose severe limitations upon cloud models used in mesoscale deposition simulations. However, within these limitations it is important to represent the physical and chemical processes which dominate the deposition process adequately. As discussed below, current models are inadequate on several counts (in particular, the treatment of entrainment and the ice phase), and thus may fail to capture important features in the deposition process. In this dissertation I describe the development and testing of a new 1.5D Eulerian cumulus cloud model which yields cloud behavior consistent with observations, and the results of an investigation of the processing of sulfate by midlatitude continental cumulus clouds under a set of realistic environmental conditions.

1.1. Historical perspective

Numerical modelling of cumulus clouds has been an active field for over 25 years. However, adequate representation of cloud properties remains a difficult problem. Dynamical processes span seven orders of magnitude and three dimensions; drop sizes may span four orders of magnitude, and all three phases of water often coexist within cloud. The problem quickly becomes intractable; one is forced to model explicitly only
the important properties for the problem at hand, and attempt to parameterize the remaining physical processes. Even this more modest goal may present a challenge when the model is used to investigate electrification or in-cloud chemistry. While a major emphasis in the past few years has been on three dimensional modelling (e.g., Klemp and Wilhelmson (1978), or Tripoli and Cotton (1982)) or the use of techniques such as nested grids (e.g., Klaassen and Clark, 1985) to achieve reasonable spatial resolution, these models quickly become real time computations, which limits their widespread use, and understanding the dynamics of such models becomes a full time effort.

Chemistry has also been included in higher dimension cloud models. Sarna (1986) examined the in-cloud conversion of \( SO_2 \) for a single case using a two dimensional Eulerian model, but did not consider different clouds or environmental parameters. Furthermore, although he simulated a midlatitude continental convective storm, the ice phase was ignored. Tremblay and Leighton (1984, 1986) have constructed a chemistry model which includes the sulfate and \( NO_2 \) cycles, and used it in conjunction with a three dimensional dynamic cloud model to investigate redistribution and deposition from small cumulus. Also, Lee (1986) has coupled chemistry and explicit warm rain microphysics in a 1.5 dimensional model investigation of fair weather cumulus.

In addition, although some work has been done in the tropics (Gilet, 1983) concerning the effects of cumulus ensembles in the deposition and redistribution processes, and for passive tracers (Niewiadomski, 1986), little has been reported on chemical evolution for midlatitude continental cumulus ensembles.

Ultimately, the aim of modeling must be to understand the three dimensional time-dependent interaction of the cloud dynamics, in-cloud turbulence, explicit microphysics and other relevant processes such as chemistry or electrification within the large scale environment. In the interim, simple models continue to play a useful role.

The limitations on computational resources dictate that most modelling studies involving secondary cloud processes such as chemistry (secondary in the sense that they normally have little effect upon the overall dynamics and microphysics) will be undertaken with relatively simple models of the dynamics and microphysics. Lagrangian parcel models (e.g., Hales, 1982); variations involving mixed and adiabatic regions (e.g., Waleck and Taylor, 1986); or versions of the Eulerian axisymmetric model of Asai and Kasahara (1967) (e.g., Lee and Shannon, 1984) have been used.

Most of these simple models are not without problems, however. Warner (1970) discussed the inability of Lagrangian bulk entrainment models to predict cloud top height and in-cloud liquid water simultaneously. Furthermore, investigations by Jensen (1986) indicate Lagrangian bulk entrainment models cannot, in general, simultaneously predict the observed in-cloud total water and a conserved thermodynamic tracer, such as \( \theta_e \) (Paluch, 1979). Cotton (1975a) compared the total liquid water calculated in a one dimensional time dependent Eulerian model based on the model of Asai and Kasahara (1967) with observations of small maritime cumulus, and found reasonable agreement when the time of predicted model values lagged the aircraft penetration by five minutes. However, no comparison of thermodynamic tracers was done.

Most of these cloud models ignore several important observations: a) that ice phase processes can be extremely important in midlatitude continental precipitation formation (e.g., Dye et al., 1986), which modifies the in-cloud aqueous chemistry, and b) cumulus clouds involve time dependent dynamic and microphysical non-linear processes, which may not couple in an obvious manner to aqueous chemistry. Furthermore, c) some important recent observations (Jensen and Blyth (1986), Rogers and Gardiner (1988), Jensen (1985), Boosman and Auer (1983), Raymond and Wilkening (1982), Telford and Wagner (1980), and Paluch (1979), for example) have illuminated the role entrainment plays in continental cumulus cloud evolution.

1.2. Observational constraints

Any cloud model results must ultimately be compared with observations. Currently, these observations are of several types: a) cloud penetrations by instrumented aircraft, b) radar reflectivity measurements, and c) ground based visual observations. The observed cloud properties have been noted and discussed by Warner (1970), Simpson (1971), Telford (1975), Cotton (1975a,b), Paluch (1979), Jensen (1985) and many others. From these, several important constraints for cumulus cloud modelling emerge:
a) Microphysical evolution

Midlatitude continental cumulus clouds have high cloud droplet concentrations (300-1000 cm$^{-3}$) and narrow initial droplet spectra (Γ ≈ 10μ, σ ≈ 2μ) with large portions of the cloud below 0 °C. These conditions are all important for the microphysical parameterization of these clouds. The high droplet concentration and narrow droplet spectra imply that the precipitation formation mechanism will likely be much different than in maritime cumulus. For clouds with temperatures below 0 °C, the ice phase processes should be considered.

Observations of isolated continental cumulonimbus (e.g., Cooper, 1978; Hobbs et al., 1986; or Dye et al., 1986) indicate the dominant precipitation formation mechanism in continental cumulus clouds is vapor deposition on ice crystals which subsequently grow into graupel particles. These in turn collect supercooled cloud droplets via riming and melt when falling through the 0 °C isotherm. Observations by Heymsfield et al. (1984) of cumulonimbus in Oklahoma also indicate that the ice phase is important in the precipitation process for clouds with moderate (i.e., droplet concentrations of ≈ 300 cm$^{-3}$) continental droplet spectra. Furthermore, significant portions of deep cumulus may have temperatures below the homogeneous nucleation temperature of water (≈ 47 °C).

b) Vertical transport

Cumulus clouds are characterized by vigorous convection. Vertical velocities may exceed 10 m s$^{-1}$ over a significant portion of the cloud lifetime for a large system. These clouds therefore represent an efficient means of vertical transport from the boundary layer. Large negative velocities are also observed and often exist simultaneously with updrafts. These downdrafts are usually driven by evaporative cooling and water loading, but recently there has been an indication that dynamic forcing (Klaassen and Clark, 1985) may also be important in some instances. In addition, shear may play an important role in organizing the convection and permitting the convection to persist by allowing falling precipitation to fall away from the updraft rather than down through it. The driving force in cumulus clouds is the buoyancy generated via latent heat release as water condenses or freezes. However, the pressure perturbation (break hydrostatic equilibrium) also modifies the velocity field, and as shown by Wilhems.

and Ogura (1972), and others, it may be of the same order of magnitude as the buoyancy.

c) Entrainment

While observed in-cloud liquid water values often show much small scale variability, no monotonic gradient is usually seen from cloud center to edge, and the transition at cloud edge is often abrupt. Also, the observed in-cloud liquid water values are generally less than expected for a non-precipitating parcel undergoing adiabatic ascent, although adiabatic cores have been reported (Jensen, 1985).

Mixing has remained one of the puzzling problems in the study of cumulus clouds. The early work on entrainment was centered on descriptions of laboratory tank experiments of thermals and plumes. These exhibit similarity flow characteristics, and lead to bulk entrainment descriptions for clouds based on the same similarity flow conditions. Warner (1970) summarized the observational evidence which indicated such an entrainment hypothesis was unsatisfactory because cloud models incorporating bulk entrainment based on similarity flow arguments were incapable of simultaneously predicting observed cloud liquid water and cloud top height. Observations presented by Paluch (1979), and others since have indicated that a two-point mixing process, i.e., mixing of cloudbase air that has risen adiabatically with environmental air at one pressure level, is sometimes the dominant mixing process. However, recent observations by Rogers and Gardiner (1986), Lawson and Rodi (1986), Jensen and Blyth (1986), and others indicate that the time dependent mixing process is often more complicated than a simple two point mixing. Mixing appears to occur at all levels of the cloud with the mixed parcels undergoing buoyancy sorting in the stages immediately prior to dissipation, as suggested by Telford (1973), and Raymond and Blyth (1986).

1.3. Purpose of this work

The goal of this work is twofold:

1. To develop a cloud model which is computationally simple, yet includes the physical processes important for chemical evolution in midlatitude continental cumuli. These processes include:
a) a microphysical parameterization which incorporates the ice phase,
b) time dependent dynamics which permits simultaneous updrafts and downdrafts to develop in-cloud, and
c) a formulation of entrainment based upon a physical parameterization.

The model will be tested against observations and examined for response sensitivities due to variation in the adjustable physical parameters.

II. To begin an examination of the dependence of the chemical fields upon the physical processes which take place in-cloud and in the near environment. Specifically I will address five points relevant to the acidification of, and sulfate deposition from, midlatitude continental cumulus clouds.

a) The relative importance of aerosol scavenging to in-cloud production of $SO_4^{2-}$ through oxidation of $SO_2$. The fraction of $SO_4^{2-}$ deposited due to sub-cloud scavenging is also examined.

b) The linearity of $SO_4^{2-}$ deposition with respect to changes in the net sulfur flux through cloudbase.

c) The importance of the entrainment of environmental air in the formation and subsequent deposition of $SO_4^{2-}$.

d) The role of ice phase microphysical processes in determining the chemical properties of precipitation.

e) The chemical effects on clouds due to a modification of the environment by previous clouds.

The first two questions concern the response of midlatitude continental cumulus clouds to variations in the chemical fields used as input. The third and fourth questions are concerned with the interaction of entrainment and ice processes and chemistry. Although important in the dynamics and microphysics for this type of cloud, their role in acidification and deposition have been studied little. The last question deals with the cloud - environment interaction in a simple case, i.e., precipitating clouds growing in an environment already processed by smaller clouds. Such a question is pertinent to the problem of the parameterization of an ensemble of cumulus clouds on a mesoscale grid.

1.4. Organisation

The cloud model developed for this investigation is discussed in Chapter 2. The base run model simulations for two clouds observed during the Cooperative Convective Precipitation Experiment (CCOPE) are found in Chapter 3. I examine the sensitivities of the dynamic and microphysical model response to variations in the adjustable physical parameters in Chapter 4. The cloud chemistry model is developed in Chapter 5. Chapter 6 is a discussion of the dependence of the chemical fields on the physical processes listed above. A summary and suggestions for future research are given in Chapter 7.
CHAPTER 2

CLOUD MODEL

2.1. General comments

The cloud model used is similar to that of Asai and Kasahara (1967) and Yano (1980) in that two concentric axisymmetrical averaged regions are incorporated in a time dependent Eulerian grid. This cumulus model departs from these models in that both regions are initially in-cloud, and interaction with the environment outside the outer cloud region is via entrainment and turbulent mixing into the outer region. This formulation was chosen as it permits simultaneous updraft and downdraft regions to develop independently in a simple dynamic framework, and furthermore allows a central core to entrain environmental air at a lower rate than either the outer region or a single region time dependent cumulus model such as used by Cotton (1975a) or Cheng (1981). Figure (2.1) illustrates the model geometry.

Because the cloud model is used for studying both shallow and deep convection, the anelastic continuity equation

\[ \frac{1}{r} \frac{\partial}{\partial r}(r \rho \nu) + \frac{\partial}{\partial z}(\rho w) = 0 \]  

(2.1)

where \( \rho_0 \) is the base state density, is used. As shown by Dutton and Fiechtl (1969), this approximation for the continuity equation permits gravity wave solutions and is satisfactory when considering deep convection. Note that a consequence of incorporating the anelastic approximation is that all processes are referenced to a constant base state density \( \rho_0 \), which allows the density to be factored in and out of the partial time derivative in the advection routines. This will be discussed in the section below on numerical advection.

In the following sections I will first write the generalized area averaged advection equations for the inner and outer regions, then discuss the terms common to all the physical parameters, i.e., the vertical and lateral eddy flux terms, dynamic entrainment, and numerical advection. The source terms for momentum, energy, and the water variables will then be derived. Appendix F is a listing of the model code.

2.2. Area averaged equations

Following Asai and Kasahara (1967), the cloud model equations are expressed in cylindrical coordinates denoted by \((r, \theta, z)\). \( r \) is the radial coordinate, \( \theta \) the azimuthal coordinate and \( z \) the vertical coordinate. The equations for momentum, energy, moisture and continuity are averaged by integrating over the appropriate cross section and dividing by \( \rho_0 \) multiplied by the cross sectional area for the regions shown in figure 2.1, to obtain the cloud model equations. Defining

![Figure 2.1](image)

**Figure 2.1**

Schematic of cloud model showing geometry of inner and outer regions.
\[ \nabla \cdot \mathbf{u} = \frac{2b}{(b^2-a^2)} \rho_0 \left( \frac{\partial \mathbf{u}}{\partial x} - \frac{a}{b} \frac{\partial \mathbf{u}}{\partial y} \right) + \frac{\partial}{\partial z} (\rho_0 \mathbf{u}) = 0 \] (2.9)

The terms on the right hand side of Eqs. 2.6 and 2.8 may be identified as: (1) vertical advection, (2) dynamic entrainment, (3) vertical eddy flux, (4) lateral eddy flux, and (5) relevant source terms. This set of equations differs from those of Asai and Kasahara (1967) and Yau (1980) by the inclusion of the vertical eddy flux terms, and a separate continuity equation for the outer region. Cotton (1975a) included the vertical eddy fluxes, but considered only one region in his calculations.

The properties of the eddy flux terms \(\nabla \cdot \mathbf{u}^w\) and \(\nabla \cdot \mathbf{u}^e\) and the dynamic entrainment parameter \(\nabla \cdot \mathbf{u}^e\) must be specified in order to close the above set of equations. Note also, that although the entrainment velocity \(\mathbf{u}^e\) is not explicitly included in the dynamic entrainment term, it is evaluated from the continuity equation in each region and used for the lateral eddy flux terms.

2.2.1. Eddy fluxes

The lateral and vertical eddy fluxes are calculated using a Smagorinsky (1963) form of the eddy diffusivity in a manner similar to Cotton (1975a). The momentum terms are given by

\[ \mathbf{w} \cdot \mathbf{w} = -2K_m \frac{\partial w}{\partial z} \] (vertical)

\[ \mathbf{w} \cdot \mathbf{u} = -K_m \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial z} \right) \] (lateral)

and the eddy mixing of scalars such as energy and moisture is given by

\[ \nabla \cdot \mathbf{u}^e = -K_m \frac{\partial e}{\partial z} \]

\[ \nabla \cdot \mathbf{u}^e = -K_m \frac{\partial 
abla \cdot \mathbf{u}^e}{\partial z} \]

where \(K_m\) is given by

\[ K_m = \frac{\kappa_A T_s}{\sqrt{2}} |D_{||}| \] (12.2)

and \(K_s = \sigma_n K_m \) \( |D_{||}| \) is the magnitude of the deformation tensor of an anelastic medium evaluated at \(r=0\) for \(\nabla \cdot \mathbf{u}^e\) at \(r=a\) for \(\nabla \cdot \mathbf{u}^e\) at \(r=a+b\) for \(\mathbf{w} \cdot \mathbf{w}\), and \(r=b\).
for $\bar{w}^2q$: $A_2$ is the buoyancy modified (Lilly, 1962) averaging length, $k_i$ has been set to 0.28 and the Prandtl number $\sigma_1 = 3.0$. The buoyancy modification to the mixing length is given by

$$A_2 = A \max\{1,(1-\sigma_1 R_i)^{0.25}\}$$

(2.23)

where $R_i$ is a generalized gradient Richardson number;

$$R_i = \frac{g(\frac{\partial}{\partial z} T_v \Gamma)}{T_v |D_{ij}|^2}$$

(2.10)

$T_v$ is the virtual temperature including water loading, defined in equations 2.32 and 2.33, and $\Gamma$ is the adiabatic lapse rate. For unsaturated air,

$$\Gamma = \frac{\kappa}{c_p}$$

(2.15)

Saturated air uses a $\Gamma$ weighted by the amounts of liquid water and ice present;

$$\Gamma = \frac{q_w \Gamma_{w} + q_i \Gamma_{i}}{q_w + q_i}$$

(2.16)

where $q_w$ and $q_i$ are the liquid water and ice mixing ratios, and $\Gamma_{w}$ and $\Gamma_{i}$ are the pseudoadiabatic gradients for water and ice, respectively;

$$\Gamma_{w} = \Gamma_{a} = \frac{1+5419.34}{1+8388504.49} \left(\frac{e_w(T)}{T_p}\right)$$

(2.7)

and

$$\Gamma_{i} = \Gamma_{a} = \frac{1+6140.91}{1+1071020.40} \left(\frac{e_i(T)}{T_p}\right)$$

(2.8)

for the saturated partial pressure of water vapor over liquid water, $e_w$, and ice, $e_i$. The temperature, $T$, is in Kelvins and the pressure, $p$, in Pascals.

This choice for the gradient Richardson number is an attempt to include the effects of the latent heat release due to condensation and freezing in the stability parameter. However, to avoid the necessity of computing the amount of freezing (or melting) water at each vertical level for this computation, all ice is assumed to have formed via sublimation. Such an approximation biases the calculated gradient $\Gamma$ toward the ice pseudoadiabatic gradient when ice is present, but the error involved is no larger than that resulting from use of the pseudoadiabatic approximation of a system containing liquid water and ice (Saunders, 1957).

$$|D_{ij}|_{r=0}$$ is given by

$$|D_{ij}|_{r=0} = \left[\left(\frac{1}{\rho} \frac{\partial}{\partial z} \omega w^2 \right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]^{1/2}$$

(2.10)

while $|D_{ij}|$, for $r=a, b(a+b)$, and $b$ is

---

**Table 2.1**

Averaging lengths for $|D_{ij}|$ evaluated at the midpoint of each region and the boundaries

<table>
<thead>
<tr>
<th>$r$</th>
<th>$A_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$(\pi a^2 d_1)^{1/3}$</td>
</tr>
<tr>
<td>a+b</td>
<td>$(2\pi a)^{1/3}$</td>
</tr>
<tr>
<td>a+b</td>
<td>$(\pi b^2-a^2)^{1/3}$</td>
</tr>
<tr>
<td>b</td>
<td>$(2\pi b^2-a^2)^{1/2}$</td>
</tr>
</tbody>
</table>
\[ \frac{|D_{ij}|}{L} = \left[ \frac{u}{\rho} \frac{\Delta p}{\Delta r} + \frac{u}{r} + \frac{1}{\rho} \frac{\partial}{\partial r} \left( \frac{\rho}{r} \right) \right] + \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 + \frac{1}{2} \left( \frac{\Delta w}{\Delta r} + \frac{\partial \tilde{u}}{\partial z} \right)^2 \right]^{0.5} \]  

(2.20)

The \(|D_{ij}|\) signifies a derivative evaluated over the same scale as \(\Delta r\). Values for other parameters in Equations 2.19 and 2.20 are given in Table 2.2.

2.2.2 Dynamic entrainment

The parameters \(\zeta_b\) at the cloud outer boundary represent the material brought into the region to satisfy mass continuity via the anelastic approximation to the continuity equation. The non-physical geometric constraints of a fixed radius at the cloud boundary result in large amounts of entrainment to satisfy mass continuity. Using the Asai and Kasahara (1967) formulation for \(\zeta_b\), i.e.,

\[ \zeta_b = \begin{cases} \tilde{\zeta} & \tilde{u}_b \leq 0 \\ \tilde{\zeta} & \tilde{u}_b > 0 \end{cases} \]  

(2.21)

results in rapid dilution of the cloud, and little growth.

Cotton (1975a) dealt with this problem in a fixed radius axisymmetric Eulerian cloud model by computing \(\zeta_b\) from the lateral and upstream vertical flux. The lateral flux is given by

\[ F_l = 2\pi R \delta z | \tilde{u}_R | \]  

(2.22)

while the upstream flux contribution is calculated from

\[ F_u = -R^2 | \tilde{w}_u | \]  

(2.23)

for the boundary at radius \(R\), and vertical grid spacing \(\delta z\). The flux weighted value of \(\zeta_b\) is then given by

\[ \zeta_b = \begin{cases} \frac{F_l + F_u}{\tilde{\zeta}_{up}} \tilde{u}_b \leq 0 \\ \frac{F_l + F_u}{\tilde{\zeta}_{up}} \tilde{u}_b > 0 \end{cases} \]  

(2.24)

where \(\zeta_b\) is the environmental value, and \(\tilde{\zeta}_{up}\) is the upstream value of \(\zeta_b\).

The rationale for such a parameterization lies in the assumption that the air in the immediate vicinity of the cloud boundary may be a mixture of cloud remnant and environmental air, rather than the unmixed environmental air. Entrainment through the cloud outer boundary will therefore be some mixture of cloud and environmental air. Although this is physically reasonable, the Cotton (1975a) flux-weighted scheme partitions the lateral and upstream contributions using the entrainment and vertical velocities. Since the entrainment velocity is calculated from \(\frac{\partial}{\partial t} w\) using the continuity equation, the flux weighting depends on \(w\) and \(\frac{\partial}{\partial t} w\), but remains a function of the fixed radius geometry.

Another approach is to base the entrainment of environmental air into the cloud on a simple physical model, and then formulate \(\zeta_b\) to reflect this hypothesis. This...

Morton (1968), and Telford (1970) discussed the importance of turbulence in the entrainment process, and proposed that the turbulence intensity is a better predictor of entrainment rate than velocities derived from similarity flow arguments. As discussed in chapter 1, recent observations (e.g., Jensen et al., 1985) show that regions of relatively undiluted cloudbase air are characterized by low turbulence levels, while mixed regions may have high turbulence levels. Figure 2.2 contains the in-cloud liquid water and turbulence level tracings made during an aircraft penetration of a moderate cumulus on July 19, 1981 during CCOPE, and indicates the difference in turbulence levels between mixed and undiluted cloud regions.

While the idea of turbulence driven entrainment is not new, little has been done to parameterize entrainment in cumulus models in terms of the turbulence intensity. Lopez (1973) presented a Lagrangian cloud model with an entrainment rate based on Telford’s (1970) proposal. In this Eulerian cloud model I have incorporated the concept of entrainment as a function of the local turbulence using a method similar to Cotton’s (1975a) flux weighted scheme. This entrainment formulation uses a weighted mean calculated from the local vertical velocity and a velocity derived from the local turbulent kinetic energy. The turbulent kinetic energy is calculated using an expression, derived by Lippa (1977), consistent with the Smagorinsky (1963) formulation of the eddy viscosity used above.

\[
TKE = 0.5 \left( \frac{k_i}{c_g} \right)^{0.07} \Delta^2 |D_{ij}|^2
\]

for a length scale \( \Delta \) (avl in the model code) and deformation tensor \( D_{ij} \) (see equation 2.20). The constant \( k_i \) is from equation 2.12, and the constant \( c_g \) from Deardoff (1973); \( k_i = 0.28 \), \( c_g = 0.7 \).

The length scale \( \Delta \) is the characteristic size of entrainment events, and will, in general, be an unknown function of cloud and environmental parameters. Turbulence spectra obtained by Rodi (1981) suggest that the dominant eddy size was approximately 200-800 meters in the Montana cumuli he observed. \( \Delta \) of this size corresponds to eddy sizes larger than the vertical resolution of the model, i.e., 100-200 meters, but smaller than the effective radial resolution (~1000 meters). This insures that the turbulent kinetic energy calculated using equation 2.25 represents the subgrid turbulent kinetic energy when evaluated at the outer boundary. The sensitivity of the model to \( \Delta \) is considered in chapter 4.

As can be seen in equation 2.25, this simple representation of the turbulent kinetic energy includes the assumption that the turbulent kinetic energy is in local equilibrium with the large scale dynamics, and as a consequence ignores the role of both advection and buoyancy generation in the local turbulent kinetic energy value. Buoyancy is included in a simple manner by using the buoyancy modified mixing length, i.e.,
\[ \Delta_{\text{雲層}} = \Delta \max (1, (1 - \sigma), \text{Ri}^{1/2}) \]

where \( \sigma \) is the turbulent Prandtl number, and \( \text{Ri} \) is the Richardson number defined in the section above.

The properties for \( \tilde{\chi} \) are determined using the formulation of Asai and Kamitani (1967), i.e.,

\[
\tilde{\chi} = \begin{cases} 
\tilde{\chi}_0 & \tilde{u}_h \leq 0 \\
\tilde{\chi}_0 & \tilde{u}_h > 0 
\end{cases}
\]

\( \tilde{\chi} \) is calculated in this manner to ensure continuity of momentum, energy, and moisture, in addition to mass, between the inner and outer regions.

### 2.2.3. Numerical advection

The area averaged equations are solved with the flux-corrected transport algorithm ETVFC (Book, Boris and Zalesak, 1981). This algorithm is conservative, maintains positivity of advected quantities for Courant numbers less than 0.5, and is characterized by low diffusion and fourth-order-accurate phase errors. The numerical scheme is a modified Lax-Wendroff method in which the vertical velocities are computed at the half time step and used to advect momentum, energy, and moisture over the full time step. This numerical algorithm has been shown (Boris and Book, 1975) to repress non-linear grid separation. Therefore all variables are defined at the same grid point, instead of a staggered grid for the velocities as is commonly used. Figure 2.3 shows the steps involved in the advection for one time step. Because the scheme is explicit, the inner and outer region equations are solved sequentially rather than simultaneously. The inner region is solved first in these computations.

The time step, \( \delta t \), is permitted to vary according to the algorithm

\[ \delta t = \min (v_\text{iso}, \max (v_\text{iso})) \]

where \( \delta z \) is the grid spacing, \( v_\text{iso} = 10 \text{ m s}^{-1} \), and \( v_\text{max} \) is the maximum velocity in the model domain during the last time step, i.e.,

\[ v_\text{max} = \max (v_\text{iso}, |v_{\text{fall}}|) \]

where \( v_{\text{fall}} \) represents the fall velocities calculated for precipitation, snow and graupel. \( v_\text{iso} \) limits the time step to a maximum amount, e.g., \( \delta t = 10 \text{ s} \) for \(|v_{\text{max}}| \leq 10 \text{ m s}^{-1} \) and \( \delta z = 200 \text{ m} \), to limit the truncation error due to a variable time step, yet permit a smaller time step if required.

---

**Figure 2.3**

Flow diagram of the advection scheme for one time step

\[ v\text{max} = \max (v_\text{iso}, |v_{\text{fall}}|) \]

where \( v_{\text{fall}} \) represents the fall velocities calculated for precipitation, snow and graupel. \( v_\text{iso} \) limits the time step to a maximum amount, e.g., \( \delta t = 10 \text{ s} \) for \(|v_{\text{max}}| \leq 10 \text{ m s}^{-1} \) and \( \delta z = 200 \text{ m} \), to limit the truncation error due to a variable time step, yet permit a smaller time step if required.
Since advection is via a flux transport algorithm, the quantities advected must be in units of density, \((\text{kg m}^{-3})\), rather than mixing ratio, \((\text{kg/kg})\). For this reason, the reference density, \(\rho_0\), is taken through the time derivative in the advection equation, and all other terms retain \(\rho\). The details of the advection algorithm are given in appendix A.

### 2.3. Momentum

The model is in essence one dimensional. Therefore only the vertical equation of motion is required. As the form for the equation in both the inner and outer regions are similar, consider only the inner region. The momentum equation is then given by:

\[
\frac{\partial}{\partial t} (\rho_0 \mathbf{w}_x) = -\nabla \cdot (\rho_0 \mathbf{w} \mathbf{w}) + \nabla \cdot \frac{\partial}{\partial z} (\rho_0 \mathbf{w}_z),
\]

(2.30)

\[
\frac{\partial}{\partial z} (\rho_0 w_z w_z) = -2 \frac{\partial}{\partial z} (\rho_0 w_z w_z) + \rho_0 \frac{\partial w_z}{\partial z} - \rho_0 \frac{\partial}{\partial z} \frac{\partial}{\partial z} w_z,
\]

(2.31)

where \(w\) is the vertical velocity (positive upwards), \(u\) is the radial component (positive outwards), and \(\rho_0\) is the density of the base state defined from the hydrostatic equation:

\[
\frac{\partial \rho_0}{\partial z} = -g \rho_0,
\]

(1.31)

for the averaged pressure \(p_0\).

#### 2.3.1. Buoyancy

\(\beta\) is the buoyancy expressed in terms of virtual temperatures \(T_v\) and mixing ratios of water vapor \(q_v\), total liquid water \(q_i\) and ice \(q_{i0}\) referenced to the density of dry air,

\[
\beta = \frac{T_v}{T_0} \left(1 - \frac{q_i + q_0}{1 + q_v}\right) = \frac{q_i + q_0}{1 + q_v},
\]

(2.32)

where the environmental value is denoted with a subscript 0, and \(T_v\) is given by

\[
T_v = T \left[\frac{1 + q_v}{1 + q_v}\right].
\]

(2.33)

#### 2.3.2. Pressure perturbation

The last term on the right hand side of equation 2.30 represents the pressure perturbation from hydrostatic equilibrium. The pressure perturbation is included in the momentum equation because of work by Cheng (1981), Yau (1979), Schlesinger (1975), Wilhelmsen and Ogura (1972), Holton (1973) and others who have shown that the pressure perturbation term is of the same order of magnitude as the buoyancy term, and is dynamically important in deep convection. I have attempted to incorporate this effect in a simple manner, following the work of Wilhelmsen and Ogura (1972), and Yau (1979). A diagnostic equation for the pressure perturbation is derived from taking the divergence of the Navier-Stokes equation, i.e.,

\[
\nabla \cdot (c_p \beta \rho_0 \nabla \psi) = -\nabla \cdot (\rho_0 u \nabla u) + \frac{\partial}{\partial z} g \rho_0 \beta
\]

(2.34)

where \(\theta_0\) and \(\rho_0\) refer to the hydrostatic base state, and \(g \rho_0 \beta\) is the buoyancy. Expanding the equation and use of the continuity equation yields:

\[
c_p \beta \rho_0 \frac{\partial^2 \psi}{\partial z^2} + \left(\frac{\partial}{\partial z} c_p \beta \rho_0 \right) \frac{\partial}{\partial z} \psi + c_p \beta \rho_0 \psi = F(r,z)
\]

(2.35)

where

\[
F(r,z) = \frac{\partial}{\partial z} \left(\frac{\partial}{\partial z} \beta - 1 \right) \left[ \frac{\partial^2}{\partial z^2} \rho_0 (u^2 + w^2) + 2 \frac{\partial}{\partial z} \left(\frac{\partial}{\partial z} \rho_0 (uw + ow + \overline{\overline{w} w}) + \frac{\partial^2}{\partial z^2} \rho_0 (w^2 + \overline{\overline{w} w}) \right) \right]
\]

To eliminate the radial derivatives in \(F(r,z)\), equation (2.35) is integrated over \(r\).

Writing \(\psi = \chi(x)\psi(x)\) and integrating from \(r_1\) to \(r_1\):

\[
c_p \beta \rho_0 \frac{\partial^2 \chi}{\partial z^2}(x) + \left(\frac{\partial}{\partial z} c_p \beta \rho_0 \right) \frac{\partial}{\partial z} \chi(x) - c_p \beta \rho_0 \chi(x) k_{on}^2 = \frac{\int \chi(x) \psi(x) \, dx}{\int \psi(x) \, dx},
\]

(2.36)

\(k_{on}\), in the last term on the left hand side of equation 2.37 is the first zero of the Bessel function \(J_0\), and occurs in the equation because of the approximation (Holton, 1973) of the radial dependance \(\psi(x)\) as a single mode of a Fourier-Bessel series. The inner region
uses $\psi(r) = J_n(k_\text{m}r)$ similar to Holton (1973), while $\psi(r) = J_n(k_\text{m}r)$ in the outer region. $n$ is chosen by requiring that the mode have a maximum near the center of the outer region is used, i.e.,

$$\frac{\partial}{\partial r} J_n(k_\text{m}r) = 0.0$$  \hspace{1cm} (2.38)

The derivative is evaluated at $r = 0.5(a+b)$ for an inner region radius of $r = a$, and an outer region radius $r = b$. A discussion of the technique used to determine the order of the Bessel mode used in the outer region may be found in Appendix B.

The integral of $F(r,z)$ must be evaluated over each region independently. Furthermore, although integration of $F(r,z)$ eliminates the necessity of knowing the radial dependence of the first derivative in equation 2.38, a radial derivative of $\bar{v}^2 \bar{\psi}$ remains, so that some assumption must be made concerning the radial dependence of $u$ and $\bar{\psi}$. Two simple assumptions are used here. The radial dependence of $u$ is calculated using the assumption that mass continuity is maintained throughout the region, i.e.,

$$u(r) = \frac{\bar{U}_a r}{a}$$ \hspace{1cm} (2.39)

in the inner region, where $\bar{U}_a$ is the velocity at radius $a$, and

$$u(r) = \left[ \frac{\bar{U}_b r}{b} \right] + \left[ \frac{a}{b} \left( \frac{\bar{U}_b r}{b} \right) \right] \frac{1}{r} = \frac{\bar{U}_b}{a} + \frac{1}{r} \left( \frac{a}{b} \right)$$ \hspace{1cm} (2.40)

for an entrainment velocity $\bar{U}_b$ at the outer cloud region radius $b$. The lateral entrainment term $\bar{\psi}^2$ is computed in an identical manner, i.e.,

$$\bar{\psi}^2(r) = \frac{\bar{U}_a r}{a}$$ \hspace{1cm} (2.41)

for the inner region, and

$$\bar{\psi}^2(r) = \left[ \frac{\bar{U}_b r}{b} \right] + \left[ \frac{a}{b} \left( \frac{\bar{U}_b r}{b} \right) \right] \frac{1}{r}$$ \hspace{1cm} (2.42)

for the outer region, where $\bar{\psi}^2_a$ and $\bar{\psi}^2_b$ are the values of $\bar{\psi}^2$ evaluated at the inner region boundary $r=a$, and the outer cloud boundary, $r=b$. These may be evaluated using an equation similar to equation 2.10, i.e.,

$$\bar{\psi}^2 = -2K_m \frac{\partial}{\partial r} u$$ \hspace{1cm} (2.43)

which yields

$$\bar{\psi}^2_a = -2K_m \frac{\bar{U}_a}{a}$$ \hspace{1cm} (2.44)

for the inner region, and

$$\bar{\psi}^2_b = -2K_m \left[ \frac{T - \Omega}{a} \frac{1}{a^2} \right]$$ \hspace{1cm} (2.45)

and

$$\bar{\psi}^2_b = -2K_m \left[ \frac{T - \Omega}{b} \frac{1}{b^2} \right]$$ \hspace{1cm} (2.46)

for the outer region. $T$ and $\Omega$ are defined in equation 2.40. $\bar{\psi}^2_a$ is evaluated in each region separately because while the lateral velocity is continuous across the boundary at $r=a$, the radial derivative of $\bar{U}_a$ is not continuous, which implies a discontinuity in $\bar{\psi}^2$ at $r=a$. As these terms only appear in the pressure perturbation equation no effort has been made to formulate a more consistent approximation for $\bar{\psi}^2$.

Equation (2.37) is solved in each region independently using a simple marching technique (e.g., Richtmyer and Morton, 1967). The boundary conditions for each region are

$$\frac{\partial}{\partial r} u = \frac{\delta \rho \partial}{c_p}$$ \hspace{1cm} (2.47)

at the lower boundary, and

$$u = 0$$ \hspace{1cm} (2.48)

at the upper boundary. The solution to equation 2.34 is then area averaged for use in the momentum equation (2.30):

$$\bar{\psi}_b = \frac{2 x_b(x^2)}{a^2} \int_0^1 r \psi_b(r) dr$$ \hspace{1cm} (2.49)

for the inner region, and
\[
\bar{\sigma} = \frac{2}{b^2a^2} \int_r^b f(r) \rho_v(r) dr
\]

in the outer region.

2.4. Energy

The total energy (per unit mass) of a system containing water vapor, liquid and ice may be approximated by

\[
\phi = H + g z + 0.5 w^2
\]

for the enthalpy \( H \) of a system containing total water \( q_{tot} = q_r + q_i + q_a \) referenced to \( 0^\circ \) C, and the density of dry air;

\[
H = (c_p z + c_w q_{tot}) T + L_v q_i - L_i q_a
\]

with the specific heats of dry air and water given by \( c_p \) and \( c_w \), and the latent heats of vaporization and freezing given by \( L_v \) and \( L_i \), respectively. The moist static energy \( H + gz \), may be expressed in terms of the potential temperature, \( \theta \), by use of the hydrometeors, i.e.,

\[
H + gz = H_d = (c_p z + c_w q_{tot}) \theta + L_v q_i - L_i q_a
\]

Again considering the inner region, conservation of energy is given by;

\[
\frac{\partial}{\partial t} \rho_a \bar{\sigma} = - \frac{\partial}{\partial x} (\rho_a \bar{w} \bar{\sigma}) + S_{\rho_a} - \frac{\partial}{\partial y} (\rho_a \bar{w} \bar{r}) + S_{\bar{\rho}}
\]

where \( S_{\rho_a} \) is the enthalpy flux due to falling precipitation given by

\[
S_{\rho_a} = \frac{\partial}{\partial z} \left[ \rho_a (\nu_p \rho_p c_s T + \left[ c_w \right] \min(T, 273.16)) \right]
\]

\[
+ L_v [v_r \rho_a + v_i q_i]
\]

for the fall velocities \( v \) and mixing ratios \( q \) of rain, snow and graupel denoted by the subscripts \( p, s, \) and \( g \), respectively. The temperature constraint is imposed on the specific heat of the ice terms to reflect an ice temperature maximum of 273.16 K.

The averaged energy, \( \bar{\sigma} \), contains the eddy terms representing the covariance of vertical velocity fluctuations, and the moisture and temperature fluctuations;

\[
\bar{\sigma} = \left( c_p + c_w q_{tot} \right) \bar{w} \bar{r} + L_v \bar{q}_i \bar{q}_a + 0.5 (\bar{w} \bar{r} + \bar{w} \bar{q})
\]

The relative importance of the terms in equation 2.56 may be estimated using the parameter values given in table 2.3. From this it may be seen that the covariance terms are of approximately the same order of magnitude as the mean velocity term. However, although \( w r \) is calculated for the eddy flux terms in the momentum equation using a mixing length approximation, computation of the moisture-temperature covariance term is not so straightforward. For this reason, and because the term is small compared to \( \bar{\sigma} \), it is not included in the expression used for the total energy, which is given by

\[
\bar{\sigma} = \Pi_d + 0.5 (w r + w q)
\]

After each time step, the temperature is calculated at each grid point, and the water vapor mixing ratio, cloud droplet mixing ratio and cloud ice mixing ratio are

<table>
<thead>
<tr>
<th>term</th>
<th>variable estimate</th>
<th>J kg(^{-1})</th>
<th>% contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_p )</td>
<td>( \bar{\sigma} )</td>
<td>200 K</td>
<td>301500</td>
</tr>
<tr>
<td>( c_w )</td>
<td>( q_{tot} )</td>
<td>0.010 kg/kg</td>
<td>12554</td>
</tr>
<tr>
<td>( L_v )</td>
<td>( q_i )</td>
<td>0.005 kg/kg</td>
<td>12505</td>
</tr>
<tr>
<td>( c_w )</td>
<td>( q_{tot} )</td>
<td>0.001 kg/kg</td>
<td>2.5</td>
</tr>
<tr>
<td>( 0.5 \bar{w} \bar{r} )</td>
<td>( \bar{w} )</td>
<td>5 ms(^{-1})</td>
<td>12.5</td>
</tr>
<tr>
<td>( 0.5 \bar{w} \bar{q} )</td>
<td>( \bar{w} )</td>
<td>3.5 ms(^{-1})</td>
<td>6.1</td>
</tr>
</tbody>
</table>
adjusted for condensation or evaporation by solving Eq. 2.57 for the enthalpy, \( e_{\text{sat}} \), inverting Eq. 2.53 using an iterative technique (discussed in Appendix C). At saturation, the vapor pressure, \( e_{\text{sat}} \), at pressure \( p \) is approximated in a manner similar to Lord, et al., (1984), to include the effects of both liquid water and ice:

\[
e_{\text{sat}}(T) = \frac{\lambda e_f(T) + \mu e_i(T)}{p - \lambda e_f(T) - \mu e_i(T)}
\]

(2.60)

for vapor saturated over liquid, \( e_f(T) \), and ice, \( e_i(T) \), with

\[
\lambda = \frac{q_l}{q_l + q_i}
\]

(2.61)

\[
\mu = \frac{q_i}{q_l + q_i}
\]

2.5. Bulk water parameterization categories

Five bulk water categories are considered: vapor (\( q_v \)), cloud droplets (\( q_c \)), precipitable water drops (\( q_p \)), cloud ice (\( q_i \)) and graupel (\( q_g \)). The water variables for the inner region are described by

\[
\frac{\partial}{\partial t} \rho \bar{x}_k = -\frac{\partial}{\partial x}(\rho \bar{w} \bar{x}_k) + \bar{w} \frac{\partial}{\partial x}(\rho \bar{w}_k) - \frac{\partial}{\partial x}(\rho \bar{w}_k q_k) - \frac{1}{n} \rho \bar{w}_k q_k \bar{w}_k + S_k
\]

(2.62)

for \( k = v, c, p, i, \) and \( g \). \( S_k \) includes the microphysical transitions between the ice water categories and the fallout terms for precipitation, snow and graupel.

2.5.1. Microphysics parameterization

The microphysical transitions include a Kessler (1969) form of autoconversion which includes a simple dependence upon the droplet concentration, the Cotton (1972) formulation of collection, and the Cheng (1981) version of the Stephens (1979) ice phase parameterization. Figure 2.3 shows the physical processes parameterized, a detailed discussion of the terms shown in figure 2.4 may be found in appendix D; however a synopsis of that appendix is presented below, while table 2.4 contains a list of the parameterized terms, and table 2.5 contains the bulk water category source terms.

The microphysics parameterization is an attempt to deal with both liquid and ice phases in a simple fashion. In order to keep the parameterization a reasonable size the Cheng (1981) version of Stephens (1979) ice phase microphysics is used rather than the more complicated South Dakota (e.g., Lin et al., 1983) microphysics scheme. The primary distinction between the parameterizations is the separation of the Stephens (1979) snow field into an ice field and snow field in the South Dakota parameterization.

The basic assumptions in the microphysical parameterization used are:
a). A monodisperse, time invariant cloud droplet population in which the total number of droplets is fixed.

b). Precipitation and graupel distributions follow a Marshall-Palmer (1948) distribution. The precipitation distribution is fixed by the intercept, while the graupel distribution is determined using a fixed total number of graupel particles.

c). The total number of snow particles is calculated using a form of the Nakanishi (1969) relationship. As discussed in appendix D, this relation has been modified to permit snow concentrations greater than 0.01 g/kg to exist in regions which have $T \leq -12^\circ$C.

d). Homogeneous nucleation occurs when the total number of snow particles exceeds $10^7$. This occurs at a temperature of approximately $-37^\circ$C.

2.5.2. Fallout terms

The fallout terms for precipitation $q_p$, snow $q_s$, and graupel, $q_g$, are calculated using

$$ S_{fall, k} = \frac{9}{2} \rho_k v_k q_k , $$

for $k = p, s, or g$. $v_k$ is the mass weighted fall velocity calculated from equation D.4 for precipitation, D.9 for snow, and D.13 for graupel.

2.6. Subcloud forcing

The cloud model is initialized with a sustained velocity forcing at cloudbase. The initial values of cloudbase pressure $p_{db}$, temperature $T_{db}$, total cloud water $q_{wb}$, and the velocity at cloudbase $w_{cb}$, are input into the model. Although, in general, actual values of these parameters exhibit some variation at cloudbase, in this model the cloudbase values of all parameters except velocity are held fixed during the forcing period.

In order to smooth the initial discontinuity at cloud base in the advective fields, a perturbation is added to the fields above cloud base at time $t=0$. The depth of the perturbation is adjustable, and set using the variable $rad_m$. The initial vertical velocity above cloud base is computed using

$$ w_{cb}(t=0) = w_{cb} \left[ 1 - \frac{z_{cb}^2}{b^2} \right] , $$

where $b$ is the outer cloud region radius, and $z_{cb}$ is the cloud base height. The moisture is initially set to a fraction of the saturated value at the environmental temperature.
\[ q_h \equiv 0, \quad t = 0 = \begin{bmatrix} 0.622 \quad \frac{e_h}{T_h} \quad (T_h - T_a(1 + 0.81) \quad \frac{p_a + \gamma_h}{p_a} \quad \frac{e_h}{e_a(T_h)} \quad \frac{e_h}{e_a(T_a)} \quad e_h \quad h \quad \text{radm} \end{bmatrix} \quad (2.63) \]

The energy is recomputed at each level using the perturbed values of velocity and moisture using equation 2.56. All model runs discussed in this dissertation use a value of radm between 0.1 and 0.3, which results in perturbed fields for about 2 to 3 km above cloud base.

The subcloud velocity forcing is accomplished by modifying the advection equations for momentum (2.30), energy (2.54), and water vapor (2.60) in both the inner and outer cloud regions to yield a sustained source of energy and moisture through cloud base for a time ptime in a manner similar to Ferrier (1987). During the perturbative time, the advection equations for the levels below cloudbase are set to

\[ \frac{\partial}{\partial t} \frac{\rho \xi}{\frac{\partial}{\partial z}} \frac{\partial}{\partial z} \rho \xi = 0, \quad (2.64) \]

by using \( \frac{\partial}{\partial z} \rho \xi \) for the source term, and setting the dynamic entrainment and entrainment fluxes to 0. The initial velocity at height \( z \) (below cloudbase) is set to \( w_c(z) \). \( (2.65) \)

<table>
<thead>
<tr>
<th>Source terms</th>
<th>PEVAP-VSEP-VSDEP-VEDEP-COND-CONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>qi</td>
<td>SMELT-AUTO-COLL-SME-CFZ-GRME-COND</td>
</tr>
<tr>
<td>qv</td>
<td>AUTO-COLL-PEVAP-PGZ-PGCOL-GRME-GPSHD</td>
</tr>
<tr>
<td>qg</td>
<td>SINIT-VSEP-SME-CFZ-SGCOL-CON-SMELT-CON</td>
</tr>
<tr>
<td>qh</td>
<td>PGZ-VEDEP-GRME-PGCOL-SGCOL-CON-GMELT-GPSHD</td>
</tr>
</tbody>
</table>

Table 2.5
Source terms for the bulk water categories

For the duration of the forcing (ptime), the sub-cloud moisture is set to the cloud base value, and the total energy is computed at each sub-cloud level using equation 2.56. At the end of the forcing period the sub-cloud moisture and energy are set to the environmental values. The response of the model to this formulation of the sub-cloud forcing will be examined in chapter 4.

2.7. Radar reflectivity equations

Radar reflectivities from the model output precipitation and graupel fields are computed using equations from Smith et al. (1975). The relations are:

\[ Z = 3.631 \left( 10^9 \right) \left( \rho_A \right)^{1.75} \left( \text{mm}^3 \text{m}^{-3} \right) \quad (2.65) \]

defined for precipitation, and

\[ Z = \frac{1.3861 \left( 10^{19} \right)}{N_{TG}} \left( \frac{\rho_A}{\rho_g} \right)^{72} \left( \text{mm}^3 \text{m}^{-3} \right) \quad (2.66) \]

for graupel. Equation 2.65 is derived assuming a Marshall-Palmer distribution with \( N_g = 8 \left( 10^4 \right) \text{ m}^{-3} \). Equation 2.66 also is derived assuming graupel follows a Marshall-Palmer distribution, but with the total number of graupel fixed (Stephens, 1979), e.g., \( N_g = 100 \text{ m}^{-2} \). \( \rho_g \) is the graupel density. If \( T \geq 273.16 \), wet hail is assumed. The reflectivity is then given by

\[ Z = 7.41 \left( 10^{18} \right) \left( \frac{1}{N_{TG}} \left( \frac{\rho_A}{\rho_g} \right)^2 \right)^{0.35} \left( \text{mm}^3 \text{m}^{-3} \right) \quad (2.67) \]

2.8. Summary

The cloud model for this sensitivity study is an axisymmetric Eulerian model based on Asai and Kasahara (1967), but which contains two cloud regions coupled to the environment via entrainment. Vertical and lateral eddy fluxes are parameterized using the Lilly (1962) buoyancy modified Smagorinsky (1963) formulation. The dynamic entrainment terms are calculated using a turbulent kinetic energy entrainment formulation developed for this model. The flux corrected transport algorithm, *ETDFT*, developed by Boris and Book (1978), is used to advect momentum, total...
energy, and the five water variables, in each cloud domain region. A simple premixed perturbation estimate using a single mode of a Fourier-Bessel series for each region of the cloud model is incorporated into the momentum equation. The interactions involving water vapor, liquid and ice are approximated by bulk water parameterizations of warm and cold rain processes. The cloud model is initiated using a sustained velocity at cloudbase.

CHAPTER 3

COMPARISON WITH OBSERVATIONS

In this chapter I will compare cloud model runs with observations of clouds made on July 19, 1981 and August 6, 1981 during the Cooperative Convective Precipitation Experiment (CCOPE) in south-central Montana. The time evolution of the model dynamic and microphysical fields are examined, and compared with aircraft and radar measurements. In addition, the base model run of July 19 will be compared with results from a published two-dimensional model simulation. Sensitivities of the cloud model to variations in the adjustable parameters will also be discussed in the next chapter.

3.1. CCOPE July 19

3.1.1. Observation summary

The cloud studied from 1616 - 1700 MDT on July 19, 1981 was a small, isolated cumulonimbus. As the temporal development of the dynamic structure and microphysics has been extensively discussed in the literature; e.g. Dye, et al., 1986 or Hebdon and Farley, 1987, the primary points will be summarized here. Figure 3.1 is a representative sounding for the day, which was characterized by weak shear (1.3 \(10^{-8}\) s\(^{-1}\)) from 800 to 400 mb and a potential instability of 1 - 2 °C as shown by the cloudbase \(\theta\). The cloudtop was observed to be approximately 6.5 km MSL (mean sea level) by the UND Citation from the time of the study initialization until about 1621 MDT, when a turret on the northeast side of the cloud began to rapidly grow. This turret reached 10.5 km MSL at around 1630 MDT. By 1650 MDT, the cloud had decayed to an anvil with falling precipitation. During the cloud study, the University of Wyoming King Air (H2) made 10 passes at approximately 6 km MSL, and the NCAR/NOAA sailplane (H9) made a spiral ascent in the cloud updraft from cloudbase to about 7 km MSL. The cloud was also observed with the NCAR CP-2 radar (10 cm) and the University of Chicago/Illinois State Water Survey CHILL radar (10 cm). Table 3.1 lists the primary
observations used in this comparison.

3.1.2. Base Model Run

The base model run was initialized with the sounding shown in figure 3.1 and the parameters listed in table 3.2. Since table 3.2 contains the variable parameters for the cloud model, the names given in the table are those used in the model listing and are defined in the list of symbols. The choice of inner and outer radii (1000 and 2500 m, respectively) for the model simulation were made on the basis of aircraft penetrations and the CP-2 RHI radar scans. The model domain was dynamically forced with a velocity of 3.0 ms⁻¹ at cloudbase in both the inner and outer regions. The cloudbase velocity in the inner region was increased to 6 ms⁻¹ at 1500 seconds in order to reduce the entrainment near cloudbase and therefore create a more nearly adiabatic core. This velocity variation is consistent with that observed near cloudbase. Forcing for the entire cloud was terminated at 2700 seconds, at which time the subcloud region was returned to the observed temperature and moisture profiles. Sensitivities of the model output to variations in the subcloud forcing are discussed in chapter 4. The cloud droplet concentration was chosen as representative of the aircraft penetrations, and yields an autoconversion threshold of 5.75 g/kg according to the relation described in

<table>
<thead>
<tr>
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<th>height temperature</th>
<th>3.9 km MSL</th>
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<tr>
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<td>q_{total} pressure</td>
<td>6.5 g/kg</td>
<td>630-635 mb</td>
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<tr>
<td></td>
<td>vertical velocity</td>
<td>1-5 ms⁻¹</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>cloudtop</th>
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<th>1618 MDT</th>
<th>1st observation</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>1630 MDT</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>radar</th>
<th>1st 5 dBZ return</th>
<th>1623 MDT</th>
<th>6.5-7.5 km MSL</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>H9 updraft observations</th>
<th>vertical velocity</th>
<th>6-7 ms⁻¹ up to 5.5 km MSL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>thermodynamics</td>
<td>nearly adiabatic</td>
</tr>
</tbody>
</table>

Miles City 1440 MDT sounding on July 19, 1981. The light dashed line represents $\theta_e = 332.55$ K, corresponding to cloudbase at 635 mb, 0.95 °C, and 6.5 g/kg. Cloudbase is from Dye, et al., 1986.
appendix D. The cloudbase values of pressure, temperature and moisture are those reported by Dye, et al. (1986) as measured by the sailplane (H9). The model run was terminated at 5500 seconds.

Figures 3.2 through 3.6 show the temporal development of the dynamic and microphysical fields for both the inner and outer regions. The model domain begins at ground level (≈ 800 m MSL). The vertical velocity (figures 3.2, 3.3) and the liquid water and ice (figures 3.3, 3.6) contour plots show a cloud which grows from approximately 6.5 km (7.3 km MSL), retains a nearly constant cloudtop height for about 1000 seconds, and then undergoes rapid growth (starting at about 2400 seconds) to final cloudtop height of approximately 10.3 km (11.1 km MSL). The peak vertical velocity is about 12 m s⁻¹. Additional plots showing the separate hydrometeor fall, entrainment velocity and turbulent kinetic energy may be found in Appendix E.

### Table 3.2
July 19 model base run parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>dz</td>
<td>200 m</td>
</tr>
<tr>
<td>arad</td>
<td>1000 m</td>
</tr>
<tr>
<td>brad</td>
<td>2500 m</td>
</tr>
<tr>
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</tr>
<tr>
<td>utm</td>
<td>1500 s</td>
</tr>
<tr>
<td>w₀</td>
<td>3.0 m s⁻¹</td>
</tr>
<tr>
<td>w₀max</td>
<td>2.0</td>
</tr>
<tr>
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</tr>
<tr>
<td>N₀ₑ</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>P₀ₑ</td>
<td>695 mbar</td>
</tr>
<tr>
<td>T₀ₑ</td>
<td>0.95 °C</td>
</tr>
<tr>
<td>q₀ₑ</td>
<td>6.5 g/kg</td>
</tr>
<tr>
<td>r₀ₑ</td>
<td>0.1</td>
</tr>
<tr>
<td>O₅ₑ</td>
<td>0.01 m⁻³</td>
</tr>
</tbody>
</table>

3.1.3. Precipitation Formation and Development

The total water plots (figures 3.3 and 3.6) and the inner region radar contour plot (figure 3.4) show a moderate intensity (45-55 dBZ, > 3 g/kg total liquid and ice), but brief (∼ 10-15 minutes) shower originating from this cloud. The total precipitation at the lowest level in the model domain was 4.9 mm rain and 0.098 mm graupel. The precipitation water (figures E.1.4 and E.1.10 in Appendix E) and graupel (figures E.1.6 and E.1.12) indicate that precipitation formation was through the ice phase. This is consistent with the observations of Dye, et al. (1986) of no raindrops or drizzle drops within the cloud. Precipitation formation first occurs around 6 km in the model domain (6.8 km MSL), again in good agreement with the CP-2 observations of the first 5 dBZ return between 6.5 and 7.5 km (MSL). No measurement was made of the actual time of formation of the observed cloud. However, a relative comparison of the model and observation is possible. Figure 3.7 shows the maximum reflectivity contours reported by Dye et al., (1986) plotted against time corresponding to the model times shown in figure 3.4. The times shown in figure 3.7 were established by setting the time of the first observed 5 dBZ return to that predicted by the model. The overall agreement is generally good, with the model predicting a higher 55 dBZ contour, and a shower of slightly shorter duration. Two 45 dBZ contours are seen in the model domain. The lower contour is in good agreement with that reported by Dye, et al. (1986). Examination of the microphysical fields indicates that the upper contour is a result of the graupel density changing from 600 kg m⁻³ to 100 kg m⁻³ due to qₑ falling below the threshold discussed in appendix D.

The rapid growth of the study cloud observed to begin at about 1621 MDT is nearly coincident with the first 5 dBZ returns from the cloud at around 1623 MDT (Dye, et al., 1986), and can be understood in terms of a combination of precipitation unloading of the updraft and glaciation. Figure 3.8 is a plot of the virtual temperature along a moist adiabat (constant θₑ trajectory), i.e., Tₑ = T(1+0.608qₑ/θₑ), and along a moist pseudoadiabat (constant θₑ); Tₑ = T(1+0.608qₑ), where qₑ is the saturation vapor mixing ratio. The curve plotted for constant θₑ corresponds to that of a parcel undergoing adiabatic expansion with no fallout of precipitation. A constant θₑ more closely approximates the virtual temperature of a parcel in which all the condensed water has fallen out. Any entrainment will result in a cooler parcel, while freezing will
increase the buoyancy and move the thermodynamic path to the right in figure 3.8.

The observed high droplet concentration (600-800 cm$^{-3}$) suggests that ascent in this cloud may be approximated initially by constant $\theta_e$ as there is no precipitation formation via drop coalescence. The subsequent formation of precipitation via ice phase processes yields both an increase in temperature (over moist adiabatic ascent) and a reduction of the water loading through fallout to produce a virtual temperature which approximately follows that shown in figure 3.8 for $\theta_e$. (The actual path is slightly different due to the additional latent heat release; $L_v/L_a \approx 1.12$).

Both the observed general cloudtop in the field of clouds around the study area on July 19 and the cloudtop of the study cloud before the rapid growth phase are consistent with the maximum height obtained ( Around 400 mb) by a nearly saturated wet parcel which started with the observed cloudbase properties. As can be seen in figure 3.5, this is also the cloudtop of the base run model cloud before the rapid growth phase. The model cloud droplet, precipitation water, snow, and graupel fields (fig. E.1.3-E.1.12 in appendix E) show that the rapid growth, which begins at approximately 2900 seconds, is coincident with the initial graupel formation (via snow aggregation and riming in the microphysical parameterization).

### 3.1.4. Entrainment

Several aircraft were used in a combined study of this cloud on July 29. The University of Wyoming King Air (H2) made 10 penetrations of the cloud between 33 and 6 km MSL (307 to 481 mb, 6 to 7 km in the model domain). The first four were through the main core of the cloud and at a constant height (6 km MSL, 381 mb). Figure 3.9 is a Paluch plot (Paluch, 1979) of the 1 Hz values from these 4 H2 penetrations (made over a 10 minute period; 1616 to 1626 MDT) using the Johnson-Wilson probe liquid water which was corrected according to the procedure described by Jensen (1985). To reduce the interpretation problems of unsaturated regions in-cloud and at the boundaries, only those points in which the 50 Hz FSSP nstrobe measurement showed drops for the entire second (the 50 Hz nstrobe measurement) and the 10 Hz FSSP liquid water was positive for the entire second are plotted. The observed in-cloud points exhibit no definite tendency towards mixing from a single location in the environment and cloudbase, but rather seem to lie in two groups; a nearly adiabatic group centered at about $6.2 \text{ g/kg}$ and 329 K, and the remaining points which have undergone various degrees of mixing.

The dashed line in figure 3.9 represents the mixing line of cloudbase air and environmental air at 421 mb. This upper limit on the observed mixing height is consistent with the maximum penetration depth of an evaporatively cooled mixture of cloudbase air and environmental air from 421 mb as may be seen in figure 3.10, which shows a saturation point mixing diagram (Betta, 1982) of air from cloudbase and the environment at 421 mb. Any mixtures of cloudbase air with environmental air from above 421 mb would be positively buoyant at 481 mb (the penetration level), and so are unlikely to penetrate to that level in the cloud. The mixtures of air from 421 mb and cloudbase air with $f \approx 0.35$ ($f$ is the fraction of cloudbase air, $(1-f)$ the fraction of environmental air) are neutrally buoyant at 481 mb.

The model output may be compared to the H2 observations keeping in mind that the model output corresponds to a time series because of the limited horizontal resolution. In addition, the limited sample volume (in space and time) of the aircraft measurements makes any direct correlation with model computations difficult. However, in this case H2 made repeated penetrations at the same altitude, which should provide some indication of the temporal evolution of convective elements at that height. From an analysis of radar and the MAPS tracks, Dye et al., (1988) reported that H2 flew through the main storm updraft during the penetrations used for comparison here. This lends support to the use of both model regions for comparison with the observed points.

Figures 3.11, 3.12 and 3.13 are a sequence of Paluch-plots showing the observed points from figure 3.9 and model points at 60 second intervals from the inner and outer regions which are saturated and have a total ice concentration < 0.1 g/kg. The time development of the air at 481 mb in these regions is shown in figure 3.14.

As seen in figures 3.11 and 3.14, the inner region first shows air which has characteristics from the levels below the penetration level and becomes increasingly closer to cloud base air, with most of the points clustering in the nearly adiabatic region at $\theta_e \approx 329$, $q_{tot} \approx 6.2 \text{ g/kg}$. The outer region points, on the other hand, show evidence of mixing from about 540 mb to 420 mb (figures 3.12 and 3.14). This mixing from the
region above the penetration level (481 mb) is also seen in the vertical velocity of the outer region (figure 3.5), which shows a downdraft of $\leq 3$ m/s$^2$ between about 6000 m and 4800 m during the mature phase before the onset of rapid growth (from about 1300 to 2600 seconds).

Detailed thermodynamic comparisons between model output and aircraft observations after about 2800 seconds are not possible because, while H2 made 6 more penetrations, the limited horizontal resolution of the model yields too rapid a glaciation of the cloud as all graupel formed in the upper levels of the cloud falls through the lower levels. The study cloud did grow in weak shear, but this was sufficient to permit only graupel fallout from the upper levels into clear air rather than into the lower regions (see Helson and Farley, 1987, fig. 3).

3.1.5. Two dimensional model comparison

The same cloud was simulated using the South Dakota 2D slab-symmetric model by Helson and Farley (1987). Initial pressure, temperature and moisture for the two-dimensional domain was the Miles City sounding shown in figure 3.1. Convective in the 19.2 $\times$ 19.2 km grid was initiated using a random temperature and moisture perturbations in the lowest three kilometers coupled with a 4.8 km diameter warm 'blob' ($\Delta T \leq 1.5$ °C) between 400 and 2000 m. The grid resolution was 200 m. Table 3.3 lists model output from the 2D model, the model discussed in this section, and some observations noted above.

The cloud base and cloud top reported for the 2D model run differ significantly from that reported by Dye, et al. (1986) and computed using the model presented here. Although there is some uncertainty in the observed cloud top (estimated $\pm$ 300 m; Dye, personal communication), the cloud-base properties were relatively well determined. From the discussion of the 2D model results, it appears no attempts were made to modify the subcloud region in order to reflect the observed cloud-base properties for their simulation. The microphysical and dynamical characteristics for both model runs are in general agreement with the observed properties; however the maximum vertical velocity in the 2D simulation is much higher than that observed.

It would appear that the major drawback of the model presented here, relative to the 2D model, is the inability to account for updraft unloading in a shear environment. Even in this weak shear case, sufficient unloading occurred to permit a liquid water updraft to coexist with falling graupel for several minutes in the 2D model; e.g., see figure 3 in Helson and Farley.
Figure 3.2  Contour plot of vertical velocity in the inner cloud region for the July 19 base run. Contour interval is 3 m s\(^{-1}\); negative velocities are shown as dotted contours.

Figure 3.3  Contour plot of the total liquid water and ice mixing ratio in the inner cloud region. Contours are from 0.01 to 3.01 g/kg; the contour interval is 0.2 g/kg. Note that the contour labels are scaled by 100.
Figure 3.4 Contour plot of the radar reflectivity (computed from equations 2.66-2.68) for the inner region. Contours are from -5 to 65 dBZ; the contour interval is 10 dBZ.

Figure 3.5 Contour plot of vertical velocity in the outer cloud region for the July 19 base run. Contour interval is 2 m/s; negative velocities are shown as dotted contours.
Figure 3.6 Contour plot of the total liquid water and ice mixing ratio in the outer cloud region. Contours are from 0.01 to 3.01 g/kg; the contour interval is 0.2 g/kg. Note that the contour labels are scaled by 100.

Figure 3.7 Contour plot of the observed radar reflectivity from Dye et al., 1986. The radar reflectivities presented in that paper have been plotted on the July 19 base run time scale. The height is plotted with respect to mean sea level (MSL) and the model domain. Reflectivities are plotted from 5 to 55 dBZ; the contour interval is 10 dBZ.
Figure 3.8 Virtual temperature plot of the July 10 sounding plotted in figure 3.1. The virtual temperature \( T_v = T (1 + 0.008(q - q_v)) \) and dew point temperature are plotted. The light dotted line is the virtual temperature for constant \( q_v \), the dashed line is the virtual temperature for constant \( \theta_v \).

Figure 3.9 Composite Paluch plot of four H2 penetrations of the July 10 cloud. Each + represents one second of data. Cloud base is indicated by the large +. The sounding from figure 3.1 is also plotted with pressures (in mb) noted.
Figure 3.10  Saturation point mixing diagram for the penetration level at 481 mb. The light dashed lines which kink at the penetration level are density isopleths. The sounding (in saturation point coordinates) is given by the solid line. The heavy dashed line shows all possible mixtures of air from cloud base (+ at 635 mb) and the environment at 421 mb. The observed points from figure 3.9 are plotted (x).

Figure 3.11  Paluch plot of model computed points for the inner region (dots) at 90 second intervals. Other points as in figure 3.9.
Figure 3.12  Paluch plot of model computed points for the outer region (dots) at 90 second intervals. Other points as in figure 3.9.

Figure 3.13  Composite Paluch plot of model computed points for the both regions (dots) at 90 second intervals. Other points as in figure 3.9.
Figure 3.14  Composite Paluch plot of model computed points for the both regions showing the time development at the penetration level. The points are shown with the model simulation time (in minutes). The inner region points are shown as dots, the outer region points by *.

3.2. COCOPE August 8

3.2.1. Observation summary

The cloud studied from 1633 - 1650 MDT on August 8, 1981 was a cumulus congestus which developed at the end of a small line of turrets. Figure 3.15 is a representative sounding for the day, which was characterized by weak shear ($\leq 1 \times 10^{-5}$ from 800 to 400 mb) and a potential instability of $1 - 2$ 'C as shown by the cloudbase $\theta_c$. The daily operation log forecast cloudtops at about 0.1 km MSL (485 mb). Cloud measurements were made by the University of Wyoming King Air (H2) flying between 504 mb and 512 mb, while the NCAR Queen Air (H6) flew repeated legs beneath cloudbase. The cloud was also observed with the NCAR CP-2 radar (10 cm).

The time series plots for the first three H2 penetrations at 500 - 512 mb are shown in figures 3.16 to 3.18. The liquid water presented is derived from the J-W measurement following the correction procedure outlined in Jensen (1985). All penetrations show a cloud colder than the environment. The vertical velocity indicates much of the cloud was descending ($u \approx -3$ to $-8$ m/s), but at least two segments (15 to 22 seconds in figure 3.16 and 50 to 60 seconds in figure 3.18) show updrafts of $\leq -7$ m/s.

The droplet concentration is around 400 to 500 cm$^{-3}$ and shows sharp cloud boundaries and nearly uniform character across much of the cloud interior. This pattern is also reflected in the liquid water measurements. The moderately high turbulence levels suggest no adiabatic core exists at the penetration level. Ice concentrations were highly variable across both penetrations and ranged from 0.0 to about 301$^{-1}$. It is interesting to note that the first and second penetrations presented show high ice concentrations in the main cloud body near the interface with a relatively strong downdraft.

H6 flew five legs beneath cloudbase over a 25 minute period and recorded peak vertical velocities of about 7 m/s (Fankhauser et al., 1983). Precipitation was reported by leg 3 (1643 MDT) and became heavy by leg 5 (1651 MDT).

3.2.2. Base Model Run

The base model run was initialized with the sounding shown in figure 3.15 and the parameters listed in table 3.4. The inner and outer radii used in the model simulation were chosen as representative dimensions from the H2 penetrations and the subcloud legs by H6. The subcloud velocity of 3.5 m/s$^{-1}$ was near the average value.
perturbation depth scaling parameter above cloudbase \((\text{radm})\) was set higher than for
the July 19 simulation, the actual perturbation depth is nearly identical as a result of
the smaller outer radius for this case (See section 2.5). The model run was terminated
at 4500 seconds.

Figures 3.19 through 3.22 show the temporal development of the dynamic and
microphysical fields for both the inner and outer regions (1000 and 1500 m, respectivley).
The model domain begins at 900 mb \((\approx 1080 \text{ m MSL})\).

The vertical velocity (figures 3.19 and 3.21) and total cloud water (figures 3.20
and 3.22) show a cloud which grows to about 5000 m (6100 m MSL, 485 mb) by 2700
seconds, with vertical velocities at the penetration level from -4 to +3 ms\(^{-1}\). Snow
(figures E.1.16 and E.1.22 in appendix E) is found at concentrations greater than
0.01 g/kg by 2800 seconds, with subsequent rapid cloud glaciation and precipitation.

| Table 3.4 |
| August 6 model base run parameters |

<table>
<thead>
<tr>
<th>model domain</th>
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</tr>
</thead>
<tbody>
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</tr>
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<tr>
<td>brad</td>
<td>1500 m</td>
</tr>
<tr>
<td>ptime</td>
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</tr>
<tr>
<td>utime</td>
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</tr>
<tr>
<td>(w_a)</td>
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<tr>
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<tr>
<td>(N_{\text{c}})</td>
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<td>(P_{\text{o}})</td>
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<tr>
<td>(T_{\text{a}})</td>
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</tr>
<tr>
<td>(q_{a})</td>
<td>7.26 g/kg</td>
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<tr>
<td>(\text{radm})</td>
<td>0.3</td>
</tr>
<tr>
<td>(\Omega_{\text{s}})</td>
<td>0.91 m(^{-3})</td>
</tr>
</tbody>
</table>

figure 3.15
Miles City 1610 MDT sounding on August 6, 1981. The light dashed line
represents \(\theta = 328.08 \text{ K} \), corresponding to cloudbase of 708 mb, 4:00 °C, 7.26
\text{ g/kg from Jensen, 1985.}

reported by H6. At 500 seconds the inner region velocity was increased to 6.3 ms\(^{-1}\).
The subcloud forcing for both regions was maintained for 3200 seconds, at which time
the subcloud temperature and moisture were returned to the observed environmental
values. The cloud droplet concentration was set at 500 cm\(^{-3}\), which yields an ice
conversion cutoff of 3.5 g/kg from the relation described in appendix D. Although

development, in agreement with observations. During the glaciation period the mean velocity increases to \( \leq +8 \text{ m s}^{-1} \). The precipitation water fields (figures E.1.13 and E.1.21) show a small amount of water (as drizzle drops) co-existing with snow and graupel from 2700 to 3000 seconds near cloudtop. The maximum cloudtop of 6500 m
(6600 m MSL, 455 mb) was achieved after the cloud glaciated. The cloud yielded a short rain shower with .91 mm of rain and .001 mm of graupel reaching the lower level in the model domain during the model run.

3.2.3. Entrainment

Figure 3.23 shows a Paluch plot of the data from the three H2 penetrations at 67 mb shown in figures 3.16 to 3.18. The points plotted are one second averages, in a second in which: a) the liquid water measured by the Johnson-Williams probe was either negative or 0.0, b) the ice concentration measured by the 2DC probe was \( \geq 2 \times 10^{-5} \), c) or the 50 Hz droplet detection (astrobe) from the FSSP indicated no droplets at any point during the second or in either the preceding or following second was rejected. The plot shows evidence for mixing from several levels, but no unmix parcel was indicated. This is consistent with the relatively high turbulence level observed in the time series of the aircraft penetrations.

Plotting the data points from figure 3.23 on a saturation point mixing diagram shows in figure 3.24 that all the observed parcels are negatively buoyant at the penetration level, but the three points near \( \theta_e \approx 324 \text{ K} \) and \( q_{a0} \approx 6.45 \text{ in figure 12 (seconds 58, 59 and 60 from figure 3.18) are near neutral buoyancy. Assuming a step mixing event, these three points are mixtures of cloudbase air and environmental air from around 570 mb. As may be seen in figure 3.26, these parcels are positively buoyant at 367 mb and would continue to rise. The other data points in figure 3.26 near high \( \theta_e \) and \( q_{a0} \), \( \theta_e \approx 323.7 \text{ and } q_{a0} \approx 6.4 \text{ (seconds 20 and 21 in figure 3.16) are consistent with mixing between cloudbase air and environmental air from around 600 mb.}

Figure 3.26 is a saturation point mixing diagram showing these points have near-neutral buoyancy at the mixing level. These points are positively buoyant at 367 mb.

Examination of the observed points with Saturation Points near 600 mb (show which have undergone the least mixing) indicates that the remaining observed data points at 511 mb are consistent with mixing between environmental air near cloud base and cloud air previously mixed at 655 mb and 550 mb. Multiple mixing events result in a large number of possible mixing fractions of the various components, and in consequence diminish the value of the Paluch analysis as a means to investigate entrainment. Clouds with well defined nearly adiabatic cores; e.g., the July 19 case discussed at the beginning of this chapter, offer a better chance of observing two point mixing, however, as may be seen in figure 3.9, even clouds with nearly adiabatic cores may have complicated Paluch diagrams.

The model output may be compared with the data points presented in figure 3.23 keeping in mind that the data points exhibit the cloud properties over a seven minute period. Figures 3.27 to 3.29 show the values at the penetration level (511 mb) predicted by the cloud model initialized with the base run parameters from table 3.3. The model points plotted are those points with \( q_e \geq 0.01 \), a total ice concentration \( \leq 0.01 \text{ g/kg and } q_e \leq 0.01 \text{ g/kg. A point is output every 60 seconds. The time development is shown in figure 3.30. As may be seen in figure 3.30, the cloud model output shows reasonable agreement with the H2 data during the later stages of cloud evolution, but prior to glaciation.

3.3. Summary

The two case studies in this chapter indicate that the model discussed in chapter 2 is able to match the general dynamic and microphysical characteristics of midlatitude continental cumulus congestus and small cumulonimbus. The aircraft observations provide detailed information on a much finer horizontal resolution than contained within the model, but repeated penetrations at the same level within a cloud offer a means of comparing the aircraft measurements with model predictions. Plotting the time series from the cloud model at the penetration level for these two clouds on conserved variable diagrams demonstrated that the computed cloud properties were in general agreement with those obtained by multiple aircraft penetrations.

The sensitivity of these results to variation in model parameters is discussed in the next chapter.
Figure 3.16  Time series of Temperature (T), corrected liquid water derived from the L-W probe, the number of ice particles (N_i), the droplet number concentration (N), the turbulence dissipation rate (e), and the vertical velocity (w) for the H2 penetration starting at 22:20 UT on August 6. The penetration level was 508 mb. The aircraft speed was approximately 80 m s\(^{-1}\).

Figure 3.17  Time series of Temperature (T), corrected liquid water derived from the L-W probe, the number of ice particles (N_i), the droplet number concentration (N), the turbulence dissipation rate (e), and the vertical velocity (w) for the H2 penetration starting at 22:30 UT on August 6. The penetration level was 512 mb. The aircraft speed was approximately 80 m s\(^{-1}\).
Figure 3.18 Time series of Temperature (T), corrected liquid water derived from the J-W probe, the number of ice particles \( N_p \), the droplet number concentration (N), the turbulence dissipation rate (e), and the vertical velocity (w) for the H2 penetration starting at 22:11:20 UT on August 6. The penetration level was 508 mb. The aircraft speed was approximately 80 ms\(^{-1}\).

Figure 3.19 Contour plot of vertical velocity in the inner cloud region for the August 6 base run. Contour interval is 3 ms\(^{-1}\); negative velocities are shown as dotted contours.
Figure 3.20  Contour plot of the total liquid water and ice mixing ratio for the inner region. Contours are from 0.01 to 3.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.

Figure 3.21  Contour plot of vertical velocity in the outer cloud region for August 6 base run. Contour interval is 2 m/s; negative velocities are shown as dotted contours.
Figure 3.22  Contour plot of the total liquid water and ice mixing ratio for the outer region. Contours are from 0.01 to 3.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.

Figure 3.23  Composite Paluch plot of the three penetrations shown in figures 3.16-3.18. The August 6 sounding from figure 3.15 is also plotted. Each + represents one second of data. Cloud base is indicated by the large +.
Figure 3.24  Saturation point diagram of data presented in figure 3.23 (i). The solid line is the sounding from figure 3.15. Also shown is a mixing line between cloud base (708 mb) and the environment at 511 mb. The density isopleths are computed for 511 mb.

Figure 3.25  Saturation point diagram of data shown in figure 3.23. The mixing line is between cloud base and the environment at 567 mb. The density isopleths are computed for 567 mb.
Figure 3.26  Saturation point diagram of data shown in figure 3.23. The mixing line is between cloud base and the environment at 694 mb. The density isopleths are computed for 694 mb.

Figure 3.27  Paluch plot of data from figure 3.23 and the model points (dots) at 60 second intervals for the inner region.
Figure 3.28  Paluch plot of data from figure 3.23 and the model points (dots) at 60 second intervals for the outer region.

Figure 3.29  Paluch plot of data from figure 3.23 and the model points (dots) at 60 second intervals for both regions.
CHAPTER 4

MODEL SENSITIVITY ANALYSIS

Since this cloud model is a coupled system of non-linear partial differential equations, an effort must be made to determine the sensitivity of the model response to changes in the adjustable parameters used in the representation of various physical processes. The response of the system to variations in the a) subcloud forcing, b) entrainment formulation, and c) microphysics parameterization will be discussed in this section. The July 19 base model run discussed in the preceding chapter is used for these studies. Table 4.1 lists the default values for the variables examined in these sensitivity studies. A full listing may be found in Table 3.2.

4.1. Subcloud forcing

Specification of subcloud forcing is one of the most difficult aspects of cumulus modelling. Usually the cloudbase thermodynamic properties and velocity, in addition to the variability and duration of forcing, are poorly known and must be estimated. All of these may influence cloud lifetime, dynamics and the microphysical development.

| Table 4.1 |
| Sensitivity run default parameter values |
| ptime | 2700 s |
| utime | 1500 s |
| wmb | 3.0 m s\(^{-1}\) |
| wmult | 2.0 |
| avl | 800 m |
| \(\theta\) | 0.50 |
| \(N_{TG}\) | 100 m\(^{-3}\) |
Cloudbase properties were well documented for the clouds discussed in chapter 3, but this examination of the effects on cloud properties due to variation in the subcloud forcing will be limited to the magnitude and duration of the cloudbase forcing.

Simulations were run for a number of cases in order to determine the relationships between these two variables and the modelled cloud properties. Tables 4.2-4.4 contain a summary of those cases considered in this section. Note that the velocity forcing studies are divided into three categories. In the first series of model runs (in table 4.3), a constant velocity of equal magnitude was imposed in both cloud regions for the entire forcing period. In order to test the effects of a spatially non-uniform cloud base forcing, the second series (table 4.3) have a cloudbase forcing of variable magnitude in the inner region coupled with a constant forcing in the outer region. The inner region velocity is increased by a constant multiplication factor (here wmult=2X) after 1500 seconds of simulation time. The third series (table 4.4) is an examination of the effect of a variation in wmult. The model runs start with the same initial velocity and vary the multiplication factor for the inner region velocity. The outer region velocity is held at the initial value during the simulation. The time at which the inner region velocity increases is set to 1500 seconds for these simulations. This corresponds to the time it takes the cloud to reach the maximum liquid water cloud top (about 6 km in the model domain). Variation of this time (stepped from 1000 to 2000 seconds) done in all simulations model runs changed the overall structure little. The results of these tests are discussed in the following sections.

4.1.1. Sensitivity to variation in forcing velocity magnitude

Table 4.2 shows a direct correlation between cloud properties and the forcing velocity magnitude. A cloudbase velocity (wcb) < 2 ms\(^{-1}\) results in very weak cloud formation confined to the first kilometre above cloudbase, but as the velocity is stepped from 2.0 to 3.5 ms\(^{-1}\), maximum cloudtop rises from 4.2 km to 10.6 km, the maximum vertical velocity increases from 4 to 15 ms\(^{-1}\), and the maximum liquid water content increases from 0.2 to >3.0 g/kg. The outer region properties exhibit similar behaviour.

Table 4.3 indicates that doubling the vertical velocity forcing in the inner region after 1500 seconds changes the overall cloud properties little. For initial velocities below 2 ms\(^{-1}\) the cloud properties are essentially unchanged. This is surprising, as a constant forcing velocity of 3.5 ms\(^{-1}\) (see table 4.2) results in a cloud growing to the tropopause (see figure 3.1), while for an initial velocity of 2.0 ms\(^{-1}\), and a step to 4.0 ms\(^{-1}\) at 1500 s, the cloudtop remains at 4200 m. In this case the increase in vertical velocity in the inner region does not yield a larger cloud for two reasons. Examination of the entrainment velocity indicates that the detrainment pattern is similar in both cases (wcb = 2.0; wmult = 1 & 2). This means that the forcing velocity increase at 1500 seconds is not sufficient to overcome the detrainment field established by the initial velocity. Furthermore, since the velocity increases only in the inner region, no protection exists for the core in the region above cloudbase. This results in rapid drying through entrainment.

In all of these cases, the maximum cloudtop is correlated with the initial vertical velocity. These results suggest that observed cloudtop regimes can be understood as the result of the interaction of the cloud dynamics and microphysics with the environment in response to different cloudbase forcings. From these preliminary tests it appears that, for this sounding, the relationships are as follows:

| Table 4.2: Variation in subcloud forcing; constant cloud base velocity |
|----------------|-------------|-------------|-------|-----|-----|
|                | w\(_{\text{cb}}\) | w\(_{\text{MAX}}\) | s\(_{\text{MAX}}\) | q\(_{\text{L}}\) \(_{\text{MAX}}\) | q\(_{\text{L}}\) \(_{\text{MAX}}\) | precip. |
| inner region   | 2.0          | 4           | 4200 | 0.2 | none | no   |
|                | 2.5          | 8           | 6200 | 2.8 | .01  | no   |
|                | 3.0          | 12          | 8900 | >3  | 1.2  | yes  |
|                | 3.5          | 15          | 10600| >3  | 1.4  | yes  |
| outer region   | 2.0          | 2           | 3800 | .01 | none | none |
|                | 2.5          | 4           | 4800 | 1.2 | none | none |
|                | 3.0          | 8           | 9600 | 2.0 | 1.2  | 1.2  |
|                | 3.5          | 12          | 10600| >3  | 1.2  | 1.2  |
e) **Cumulonimbus**

Once the initial velocity is around 3 ms\(^{-1}\), sufficient moisture is available at the -10 to -20°C temperature interval to enable the ice phase to dominate the overall cloud structure. This results in precipitation and an increased cloud depth due to additional latent heat release coupled with updraft unloading.

Photographs (taken from H5) of the general cloud field on July 19 indicate that many of the clouds were near the liquid water cloud top, while a few (of which the study cloud was an example) grew to the cloud top shown for forcing around 3 ms\(^{-1}\). Therefore, the fact that these model simulations indicate that the cloud properties are highly sensitive to threshold velocities, i.e., those corresponding to small 'fair weather' type cumulus, cumulus congestus, and cumulonimbus, may be reflected in the observed cloud field.

### 4.1.2. Sensitivity to inner region velocity variation

Table 4.4 lists the cloud properties for a variation in the inner region velocity multiplication factor (\(\text{wmult} \) in table 3.2). All of these cases had cloud top above the liquid water cloud top, and precipitated. The maximum snow content was greater than 2 g/kg in all cases. The total precipitation also varies by an order of magnitude, but shows a maximum near \(\text{wmult} \approx 1.5 \) to 2.0. This shows some correlation with the maximum velocity in the inner region. Although the outer region does not show much variation in the maximum liquid water content, the maximum snow content varies by an order of magnitude, and is correlated with \(\text{wmult} \).

These processes are interrelated. Figure E.2.1 in Appendix E shows the inner region velocity for \(\text{wmult} = 0.5, 1.5 \) and 2.5. Although the maximum velocity in each case is 9 ms\(^{-1}\), the duration and vertical extent of the 9 ms\(^{-1}\) interval is directly correlated with \(\text{wmult} \). Examination of figure E.1.1 (the entrainment velocity for the base run case) indicates that when the velocity is increased to 6.0 ms\(^{-1}\) (\(\text{wmult} = 2 \)) the entrainment near cloud base approaches zero. This is a result of the mass flux through cloud base almost exactly offsetting the entrainment required to satisfy mass continuity (see eqn. 2.7) caused by the buoyancy acceleration (which yields a vertical gradient in \(w\)). Dropping the inner region velocity (\(\text{wmult} = 0.5 \)) results in more entrainment and a rapid decrease in the vertical velocity. This decrease in vertical velocity results in a
4.1.3. Subcloud forcing summary

Overall, it appears that the cloud properties (cloudtop height, maximum vertical velocity, precipitation, etc.) are highly sensitive to the subcloud forcing. These tests indicate that for the range of inner region velocities close to the value necessary for reducing the entrainment near cloud base, the cloud properties are close to that observed for the cumulonimbus on July 19 (see Chapter 3). This range of velocities (\(w_{\text{m}} \approx 1.5 \text{ to } 2.0, \ w \approx 4.5 \text{ to } 6.0\)) corresponds to the upper range of vertical velocities observed below cloud base for that case.

4.2. Entrainment formulation

Table 4.5 lists cloud properties for model runs using different parameterizations of the entrainment with the base run parameters found in Table 4.1. The entry labeled A&K uses the entrainment assumption of Asai and Kasahara (1967), i.e., that the entrainment necessary to satisfy mass continuity is lateral (see eqn. 2.21). The entry labeled Cotton denotes use of Cotton’s (1975) flux-weighted entrainment hypothesis based on the values of vertical and entrainment velocity (eqn. 2.24). The remaining entries denote the value of the mixing length (in meters) used in the entrainment formulation developed in Chapter 2. This weighs the lateral entrainment by the value of the local turbulent kinetic energy (eqn. 2.25).

A comparison of the dynamic, microphysical and thermodynamic properties resulting from these different entrainment formulations will be found in the following sections.

4.2.1. Sensitivity of dynamics and microphysics to entrainment formulations

Table 4.5 shows that the general characteristics of the model simulations are relatively insensitive to the entrainment formulation, the exception being the run incorporating the Asai and Kasahara (1967) form. This entrainment formulation results in a cloud with lower updraft velocity in both the core and outer cloud region, much less snow in both regions, and no precipitation. The Cotton (1975) and turbulent kinetic energy formulations show good agreement in these gross characteristics. The turbulent kinetic energy formulation also shows a strong correlation between the mixing length (m) and the total precipitation, outer region velocity, liquid and snow mixing ratios.
### table 4.5
variation in entrainment formulation

<table>
<thead>
<tr>
<th>entrainment</th>
<th>$w_{MAX}$</th>
<th>$q_{MAX}$</th>
<th>$q_{g MAX}$</th>
<th>$q_{g MAX}$</th>
<th>precip. rain/grau pel mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ms$^{-1}$</td>
<td>m</td>
<td>g/kg</td>
<td>g/kg</td>
<td></td>
</tr>
<tr>
<td>inner region</td>
<td>A&amp;K</td>
<td>8</td>
<td>10000</td>
<td>&gt;3</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Cotton</td>
<td>12</td>
<td>10200</td>
<td>&gt;3</td>
<td>&gt;3</td>
</tr>
<tr>
<td></td>
<td>600 m</td>
<td>12</td>
<td>10500</td>
<td>&gt;3</td>
<td>&gt;3</td>
</tr>
<tr>
<td></td>
<td>800 m</td>
<td>12</td>
<td>10800</td>
<td>&gt;3</td>
<td>&gt;3</td>
</tr>
<tr>
<td></td>
<td>1000 m</td>
<td>9</td>
<td>10200</td>
<td>&gt;3</td>
<td>&gt;3</td>
</tr>
<tr>
<td>outer region</td>
<td>A&amp;K</td>
<td>4</td>
<td>9000</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Cotton</td>
<td>9</td>
<td>10200</td>
<td>&gt;3</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>600 m</td>
<td>9</td>
<td>10200</td>
<td>&gt;3</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>800 m</td>
<td>8</td>
<td>10200</td>
<td>2.0</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>1000 m</td>
<td>6</td>
<td>10000</td>
<td>2.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Maximum cloudtop is nearly independent of these parameters. (see figures E.22-E.23 in Appendix E)

4.2.2. Sensitivity of thermodynamic fields to entrainment formulations

Differences in the computed cloud properties do appear when the model output is plotted on a Paluch diagram. Figures 4.1 through 4.5 are plots of the model output from both regions at the H2 penetration level (481 mb; 5200 m in the model domain). The observed data points discussed in section 3.1.4 are also plotted. Note that the limits regarding precipitation and ice in the computation of $\theta_i$ imply that the model points are those prior to substantial glaciation (ice less than 0.01 g/kg) in the cloud, and $w_{MAX}$ are representative of the initial cloud growth and liquid cloud dwell phase.

Figure 4.1 is the Paluch diagram in which the model results for the Assai and Kasahara entrainment formulation and the observed points described in Chapter 2 are compared. This diagram shows an inner model region (at 481 mb) which has undergone more mixing than observed (the cluster of points near $\theta_i \approx 329$ and $q_{MAX} \approx 0.2$ g/kg). The outer model region (the points near $\theta_i \approx 326$ and $q_{MAX} \approx 2.5$ g/kg) exhibits some mixing from near the penetration level. Figure 4.2 shows the Cotton (1975) formulation. In this case, both the inner and outer regions show little mixing, and most of the computed points lie close to the nearly adiabatic values observed and discussed in Chapter 3. Paluch diagrams at the same level for model simulations using the turbulent kinetic formulation with $\text{AVL} \approx 600, 800, \text{and 1000 meters}$ are shown in figures 4.3, 4.4 and 4.5. All of these show inner region points which agree reasonably well with the observations of the nearly adiabatic points (the points near $\theta_i \approx 326$ and $q_{MAX} \approx 6.5$ g/kg), and show mixing in the outer model region (the points near $\theta_i \approx 326$ and $q_{MAX} \approx 3$ g/kg) from near the penetration level. The turbulent kinetic energy formulation tends to emphasize entrainment (relative to the Cotton formulation) in regions of large gradients of velocity or buoyancy, or in regions of high shear (see equations 2.20, 2.25, and 2.26); e.g., at cloudbase or near cloud top, or at updraft/downdraft interfaces, while the Cotton formulation responds only to the ratio of $\partial \omega / \partial z$. The turbulent kinetic energy formulation includes an additional parameter, i.e., the mixing length AVL, but these tests indicate that the model results are not extremely sensitive to the choice of mixing length. Both the base run for July 19 and August 6 were run with AVL $\approx 0.5$ (rad - arad).

4.3. Sensitivity to microphysical parameterization

The basic microphysical parameterization has been well documented in the literature; e.g., Stephens (1979), Cheng (1981), or Taylor and Baker (1987). In the clouds under consideration precipitation is initiated primarily through the ice phase. Therefore, this series of sensitivity tests is focused on the impact of variation of the important parameters in the ice portion of the microphysical parameterization. In particular, I will examine the effect of a variation of the adjustable parameters controlling the snow and graupel distributions.
Figure 4.1 Combined Paluch diagram for the inner and outer cloud regions using the Asai and Kasahara entrainment formulation. Cloudbase is denoted by the large +, H2 observations (1 sec. averages - see Chapter 3) are denoted by the small +, and the model computed points (at 60 second intervals) by the filled circles.

Figure 4.2 Combined Paluch diagram for the inner and outer cloud regions using the Cotton entrainment formulation. Cloudbase is denoted by the large +, H2 observations (1 sec. averages - see Chapter 3) are denoted by the small +, and the model computed points (at 60 second intervals) by the filled circles.
Figure 4.3 Combined Paluch diagram for the inner and outer cloud regions using the TKE entrainment formulation with \( \text{eef} = 600 \text{ m} \). Cloudbase is denoted by the large +, H2 observations (1 sec. averages - see Chapter 3) are denoted by the small +, and the model computed points (at 60 second intervals) by the filled circles.

Figure 4.4 Combined Paluch diagram for the inner and outer cloud regions using the TKE entrainment formulation with \( \text{eef} = 800 \text{ m} \). Cloudbase is denoted by the large +, H2 observations (1 sec. averages - see Chapter 3) are denoted by the small +, and the model computed points (at 60 second intervals) by the filled circles.
4.3.1. Sensitivity to snow concentration

4.3.1.1. Snow particle number

The total number of particles in the snow category is defined as a function of temperature and the total snow concentration, i.e.,

$$N_{TS} = \max \left[ \frac{O_{NS}}{\rho_{C} \left( T_{50} \right)^{0.5}}, \frac{1}{\rho C_{T}} \right] \left[ \text{m}^{-3} \right].$$ (4.1)

where the first term on the right hand side of equation 4.1 is the Fletcher (1969) curve, and the second term limits the maximum snow crystal mass to 0.001 g. This second term is included to alleviate the tendency of the basic Stephens (1979) parameterization to artificially force snow into the graupel category at temperatures above around -12°C, which results from limiting the maximum snow crystal mass to 0.001 g, and computing the number of snow particles by the Fletcher curve. Simultaneous constraint of both these parameters yields a maximum snow mixing ratio permitted (a function of temperature), in the Stephens parameterization any snow mass in excess of the maximum snow mixing ratio is dumped into the graupel field. Using these criteria, the critical snow mass is reached for a concentration of 0.01 g/m³ at T ≈ -12°C. The result of this set of conditions is rapid conversion of snow to graupel at the temperature interval where snow first forms, leading to a rapid glaciation of the cloud due to graupel riming. This is a consequence of including ice crystals and snow in the same bulk category, and while a parameterization which distinguishes them (such as a version of the South Dakota scheme; e.g., Lin et al., (1983), Rutledge et al., (1986), or Ferrier (1987)) is possible, such a scheme introduces another set of fields, microphysical transitions and assumptions (such as the snow size distribution and the ice particle concentration). The form for $N_{TS}$ given in equation 4.1 (and in equation D.6) is an attempt to decouple the number of snow particles from the exponential character of the Fletcher relation at temperatures above about -12°C during the later stages of cloud evolution. This form for $N_{TS}$ is used in all model results presented here. In addition, the snow to graupel transitions have no excess snow mass term as is used in Stephens (1979), or Cheng (1981). Note, however, that the parameterization does include a snow aggregation term and an excess snow riming term in the snow to graupel transitions (see tables 3.4 and appendix D).
\[ v_s = -\frac{130}{6} \Gamma(4.5) \left[ D_p \right]^{0.5} \]

**c) \( q_s \) - Snow**

Snow crystals are assumed to be formed according to a temperature dependent function based upon Fletcher's (1969) observation that the number of ice crystals increased exponentially with temperature, i.e.,

\[ N_{TS} = \max \left( 0.01 \times e^{(273.15 - T) / 10^3}, \rho_s / \rho_a \left( 10^3 \right) \right) \text{ m}^{-3} \]

The first term on the right hand side of equation (D.6) is the form given by Fletcher (1969) for the ice particle concentration as a function of temperature, while the second term limits the maximum snow crystal mass to 0.001 g. As discussed in section 4.11, this second term is included in \( N_{TS} \) to alleviate the tendency of the basic Stephens (1979) parameterization to artificially force snow into the graupel category at temperatures above around \(-12^\circ C\). Such a situation results from limiting the maximum mass crystal mass to 0.001 g, and computing the number of snow particles by the Fletcher relation. The form for \( N_{TS} \) given in equation (D.6) decouples, albeit in a simple fashion, the number of snow particles from the exponential character of the Fletcher relation at temperatures above about \(-12^\circ C\) during the later stages of cloud evolution. This form for \( N_{TS} \) is used in all model results presented here.

The diameter and fall velocity for snow are computed using the expression from Stephens (1979). Three mass regimes are considered:

\[ D_c = \begin{cases} 
16.286 \left( M_s \right)^{0.5} & M_s < 10^{-10} \\
6.072 \left( M_s \right)^{0.5} & 10^{-10} \leq M_s \leq 10^{-8} \\
1.585 \left( \frac{M_s}{10^8} \right)^{0.27} & M_s > 10^{-8} 
\end{cases} \]

for a snow particle mass

\[ M_s = \frac{q_s \rho_s}{N_{TS}} \]

and

\[ v_s = -\frac{100000}{p} \left( \frac{M_s}{10^8} \right)^{0.5} \]

\[ v_s = \begin{cases} 
304 \cdot D_c & M_s < 10^{-10} \\
1250 \cdot D_c & 10^{-10} \leq M_s \leq 10^{-8} \\
4.838 \cdot D_c^{0.25} & M_s > 10^{-8} 
\end{cases} \]

where the initial term on the right hand side is a correction for the fall velocity at pressure \( p \).

**d) \( q_g \) - Graupel**

Graupel particle diameters are computed using the Stephens (1979) mass categories: light graupel \( (M_s < 10^{-6}) \), heavy graupel \( (10^{-6} \leq M_s \leq 2.0 \times 10^{-5}) \), and hail \( (M_s \geq 2 \times 10^{-5}) \), for

\[ M_s = \frac{q_g \rho_s}{N_{TG}} \]

\( N_{TG} \) is the total number of graupel particles, and is held constant. The particle diameter is given by:

\[ D_g = \left[ \frac{\left( \frac{q_g \rho_s}{\pi N_{TG} \rho_g} \right)^{0.333}}{\pi N_{TG} \rho_g} \right]^{0.333} \]

where \( \rho_g \) is the graupel particle density;

\[ \rho_g = \begin{cases} 
100 & M_s < 10^{-6} \\
600 & 10^{-6} \leq M_s \leq 2.0 \times 10^{-5} \\
900 & M_s \geq 2 \times 10^{-5} 
\end{cases} \]

The mass weighted fall velocity is given by

\[ v_g = -\frac{\Gamma(4.5)}{6} \left( D_g \right)^{0.5} \]

where
\[
\kappa_{eq} = \begin{cases} 
59.4 \left( \frac{100000}{p} \right)^{0.5} & \rho_e = 100 \\
4 \frac{\rho_e}{3 \rho_0} \left( \frac{\rho_e}{\rho_0} \right)^{0.5} & \rho_e \geq 600
\end{cases}
\]

\(\kappa_{eq}\) for \(100 < \rho_e < 600\) is the maximum of the two functions calculated in equation D.14.

Source terms

The microphysical parameterization terms discussed in Chapter 2 (see Figure 23, Table 2.4 and Table 2.5) are presented below. The computer code for these terms is found in subroutines PARKES and PARICE in Appendix F. Unless noted otherwise, all source terms in this section have units of kg/kg (air) s\(^{-1}\).

D.1 Liquid phase processes

AUTO - autoconversion

This term computes the amount of cloud water, \(q_c\), converted to precipitation water, \(q_p\), via coalescence of cloud droplets. The form used is the Kessler (1969) formula:

\[
\text{AUTO} = \begin{cases} 
\kappa (q_c - q_{eq}) & q_c > q_{eq} \\
0 & q_c < q_{eq}
\end{cases}
\]

where \(\kappa = 10^{-3}\) s\(^{-1}\) and \(q_{eq}\) is usually specified in terms of a limiting concentration m\(^{-3}\), i.e., around \(0.001\) kg/kg, \(q_{eq}\).

Cotton (1972) computed a time dependent form for the autoconversion constant with the Kessler (1969) precipitation collection term. Unfortunately, this autoconversion relation was a rather complicated numerical fit to the residual of the stochastic collection equation and the precipitation collection term for two droplet concentrations (100 cm\(^{-3}\) and 300 cm\(^{-3}\)). In addition, while the time dependence of autoconversion is well defined for a Lagrangian parcel, a direct extension to an Eulerian grid is not straightforward.

At low droplet concentrations and relatively high liquid water values, the Kessler autoconversion term is a reasonable match for the time average values of the rates computed by Cotton. However, the Kessler form overpredicts for low liquid water concentrations. I have used a modified form of the Kessler form for autoconversion which incorporates, albeit in an intuitive fashion, the main points of the Cotton formulation, i.e., the dependence on droplet concentration, and an allowance for the time dependence of initial droplet coalescence. The Kessler form is used rather than the autoconversion formula derived from the stochastic collection equation by Berry (1968). This is because, as discussed by Cotton (1972), Berry’s formulation apparently does not distinguish between autoconversion and accretion and so yields an excessive autoconversion rate. The Orville and Kopp (1977) modified version of the Berry equation also gives autoconversion rates larger than those computed by Cotton (1972) from the stochastic collection equation.

While \(\kappa\) and \(q_{eq}\) may be readily determined for maritime (\(\approx 100\) cm\(^{-3}\)) and moderate continental droplet concentrations (\(\approx 300\) cm\(^{-3}\)), the actual values of \(\kappa\) and \(q_{eq}\) appropriate for use in midlatitude continental clouds are not well established. Aircraft observations of these clouds (e.g., Dye et al. (1986), Jensen (1985), or Dye et al. (1983)) clearly show instances in which the cloud liquid water content is above 4 g/kg (i.e., \(\approx 2.5\) g m\(^{-3}\) at 500 mb), and no precipitation or drizzle size drops are observed. These clouds typically have droplet concentrations around 600 – 1200 cm\(^{-3}\). In such clouds, use of \(q_{eq}\) \(\approx 1\) g/kg is inappropriate and will lead to precipitation formation via coalescence rather than through the ice phase processes.

I have modified the Kessler autoconversion form to include a dependence upon droplet concentration, and to crudely approximate the time dependent behavior observed in the numerical solutions presented by Cotton (1972). The dependence upon droplets concentration is estimated from the observation of no coalescence at high concentrations, and from the time-averaged rates at low-to-moderate droplet concentrations computed from the equations presented in Cotton (1972). (Note that \(b'\) and \(a'\) given in equation 19 of the above paper are inconsistent with the values presented in
figure 4 of that paper; \( \alpha' = e^{3.0} \text{ m}^{-1.75} \text{ m} \) and \( \alpha' = 2.847 \times 10^{4} \\text{ m}^{-0.5} \) offer better agreement with the data presented in figure 4.) To reduce the impact of the autoconversion term for clouds with large droplet concentrations, \( q_{\text{w}} \) is calculated using a simple linear expression:

\[
q_{\text{w}} = 7.5 \times 10^{-6} N_{\text{cr}} - 2.5 \times 10^{-4} \quad [\text{kg/kg}],
\]

where \( N_{\text{cr}} \) is the droplet concentration \( \text{cm}^{-3} \). Note that in this expression the limit on the autoconversion in determined by the liquid water mixing ratio and not the liquid water concentration (as is usually the case). \( q_{\text{w}} \) rewritten this way provides a means to distinguish a low liquid water situation \((\rho_{w} \leq 1.5 \text{ g/m}^3)\) in a maritime or moderate continental cloud from the case of high droplet concentration, high mixing ratio, as may occur well above cloudbase in a continental cloud. In both instances, the total liquid water concentration \((\text{kg/m}^3)\) may be the same due to the air density dependence upon height in the atmosphere. However, while autoconversion in probable for the maritime or moderate continental case, given sufficient time, it is unlikely to happen in the continental case. Even though these cases are degenerate with respect to total liquid water \((\text{kg/m}^3)\), a clear distinction exists with respect to liquid water mixing ratio \((\text{kg/kg})\).

\( \kappa \) has not been modified.

COLL - Collection

This term represents the collection of cloud droplets by precipitation (accretion). Collection is determined using a continuous collection model and a Marshall-Palmer (1948) distribution for the precipitation. From these one may show:

\[
\text{COLL} = \frac{130\pi}{4} \Gamma(3.5) \left[ \frac{6}{\lambda_{\kappa} + T(4)} \right]^{2/5} q_{\text{w}} N_{\text{cr}}^{12} E_{\kappa} \left( \rho_{\kappa} \right)^{2/5},
\]

where \( N_{\text{cr}} \) is the intercept of the precipitation distribution, i.e., \( N(0) = \frac{N_{\text{cr}}}{10} \exp\left( -12 \right) \), and \( E_{\kappa} \) is the collection efficiency modeled on potential flow about the raindrop.

\[
E_{\kappa} = \frac{S_{a}^{2}}{S_{a}^{2} + 0.5}.
\]

where (Stephens, 1979),

\[
S_{a} = \frac{1000 (\tau_{a} N_{a}^{2/5})}{9.0 \mu (\rho_{a} / \rho_{b})^{2/5}}\left[ \frac{v_{s}^{3}}{D_{b}^{2}} \right].
\]

\( \mu \) is the dynamic viscosity of air (see equation D.47), and \( v_{s} \) is the precipitation fall velocity (equation D.5).

PEVAP - The evaporation of precipitation

Evaporation of precipitation drops is allowed to occur in subsaturated regions using:

\[
\text{PEVAP} = 2\pi N_{\text{cr}} \left[ 1 - \frac{q_{\text{w}}}{\rho_{b}} \right] G_{w}(T; p) \left[ 1 + 0.265 \frac{\rho_{b}}{\mu} \frac{|v_{s}|}{D_{b}} \right] \frac{1}{\rho_{a}},
\]

for a saturation vapor mixing ratio \( q_{w} \). \( G_{w}(T; p) \) is given by:

\[
G_{w}(T; p) = \left[ \frac{L_{w}^{2}}{K_{T} R_{g} T^{4}} \left( \frac{R_{g} T}{\psi_{w}(T)} \right)^{2} \right],
\]

where \( K_{T} \) is the thermal conductivity of air (see equation D.49), \( R_{g} \) is the gas constant for water vapor, and \( \psi \) is the molecular diffusivity of air (see equation D.46). PEVAP is derived in a manner analogous to the Stephens (1979) derivation of the ice vapor deposition terms.

D.2 Ice phase processes

Three regimes exist for the ice phase processes; at \( T = 273.16 \text{ K} \), most ice terms become activated, and water and ice co-exist in the cloud until the homogeneous nucleation temperature of liquid water, i.e., \( \approx -35^\circ \text{C} \), is reached. The ice phase terms will be considered according to temperature.

D.2.1 \( T < 273.16 \text{ K} \)

Since several terms are set to zero at the homogeneous nucleation temperature, any change resulting when the temperature falls below this threshold will be noted individually.
SINIT - snow initiation

For a positive supersaturation with respect to ice (Stephens, 1979):

\[ SINIT = \frac{1}{\delta_m} \min \left( \frac{1}{\rho_0} M_{ai} N_{TS}, \left( q_s - q_a \right) \right) \]

where \( M_{ai} = 1.0 \times 10^{-12} \) kg is the individual ice crystal mass, and \( N_{TS} \) is the total number of snow particles (equation D.6). \( q_s \) is the saturation vapor mixing ratio with respect to ice. \( \delta_m \) is the time step for the microphysical processes, and is set to

\[ \delta_m = \max \left( 10 \ , \ \delta \ \right) \ \text{[seconds]} \]

for \( \delta \) defined in equation 2.28. The second term in equation D.22 ensures that the ice crystal production is limited by the available water vapor. In addition, homogeneous nucleation is assumed for \( N_{TS} \geq 10^7 \). This occurs at about -37 °C.

If the air is unsaturated with respect to ice, \( SINIT = 0 \).

SRME - riming of snow

Riming of snow is calculated using an equation similar to the collection term defined in equation D.3. Collection efficiencies calculated by Pitter (1977) and Schatz et al., (1975) indicate that droplets with diameters < 10 μm have extremely low probability of being collected. Therefore, if the cloud water diameter, \( D_s > 0.00001 \) and \( q_s > 5 \times 10^{-9} \),

\[ SRME = \min \left[ \frac{N_{TS} \pi D_s^2}{4} \left( \frac{v_s}{E_{v,v}} q_s \right) \delta_m \right] \]

where \( D_s \) is the snow particle diameter (equation D.7), \( v_s \) is the snow fall velocity (equation D.9), and \( E_{v,v} \) is the collection efficiency calculated identically to \( E_{v,v} \) in equation D.18, with

\[ S_{tk} = \frac{1000 D_s^2}{9 \mu D_s} \]

\( q_s \) is the amount of water remaining after autoconversion and collection, i.e.,

\[ q_s = q_s - (\text{AUTO + COLL}) \delta_m \]

If \( q_s \) or \( q_a \) are outside the bounds noted above, or if \( N_{TS} \geq 10^7 \), SRME = 0.

CSFZ - freezing of cloud drops to snow

This term accounts for the homogeneous freezing of cloud drops:

\[ CSFZ = \begin{cases} \frac{q_s}{\delta_m} & N_{TS} \geq 10^7 \\ 0 & N_{TS} < 10^7 \end{cases} \]

where \( N_{TS} \) is the total number of snow particles calculated from equation D.6.

SMELT - melting of snow

For \( T < 273.16 \)K no melting of snow is allowed, i.e., SMELT = 0.

PGFZ - freezing of precipitation

Freezing of precipitation via snow crystal contact is calculated using the expression of Stephens (1979):

\[ PGFZ = \pi N_{TS} D_s^2 \left( \frac{3.1875 v_s - v_s}{q_s} \right) \]

If \( N_{TS} \geq 10^7 \), PGFZ = \( \frac{q_s}{\delta_m} \).

GRME - riming of graupel

Riming is calculated whenever \( q_s > 5 \times 10^{-9} \) and \( D_s > 10^{-5} \). The expression is similar to that used for snow riming:

\[ GRME = \min \left[ \frac{4 \pi}{9} E_{v,v} N_{GR} \left( \frac{v_s}{D_s^2} q_s \left( \frac{q_s}{\delta_m} - \text{SRME} \right) \right) \right] \]

where the collection efficiency, \( E_{v,v} \), is calculated using equation D.18 with

\[ S_{tk} = \frac{405 D_s^2}{9 \mu D_s} \]

\( N_{GR} \) is the total number of graupel particles, and is fixed in this parameterization. \( D_s \) is the graupel particle diameter (equation D.11), \( v_s \) is the graupel fall velocity (equation D.13), and \( q_s \) is defined in equation D.26. The second term in equation D.29 limits
graepe-l riming to the available cloud water.

GRME = 0 for \( N_T \geq 10^7 \).

GMELT - melting of graepe-

GMELT = 0 for \( T < 273.16K \).

PGCOL - graepe collection of precipitation

The collection term is from Stephens, (1979)

\[
PGCOL = \pi | v_p - v_s | q_p \frac{N_{TG}}{0.5 D_k^2 + 2 D_k D_s + 5 D_s^2}
\]

(D.2)

PGCOL = 0 for \( N_T \geq 10^7 \).

SGCOL - graepe collection of snow

The snow collection term is from Cheng, (1981), and uses a dry collection efficiency of 0.1;

\[
SGCOL = \frac{3\pi}{7} 0.10 N_{TG} | v_s | q_s D_k^2
\]

(D.12)

SGCON - conversion of snow to graepe

\[
SGCON = \left[ 0.01 e^{0.07(T - 273.16)} \max (0, q_s - 0.0001) \right] + \left[ \frac{q_m}{\delta m} \right] \text{Heav}(q_s - 0.0001) \text{Heav}(q_m - 0.0001)
\]

(D.13)

The first term on the right hand side is the aggregation term of Cheng (1981). The second term limits the snow growth rate via riming in high liquid water situations (Ruddle and Hobbs, 1983). \( \text{Heav} \) is the Heaviside function centered at 0.

GPSHD - precipitation shedding due to wet growth of graepe-

Wet growth is calculated using the expression of Cheng, (1981). The rate at which water can freeze on the graepe is;

\[
PGWG = \max \left( 0, -2 \pi N_{TG} F_s D_k \rho_0 L_v \left( q_s - q_{sw} \right) + K_T \left( T - 273.16 \right) \frac{\rho_0}{L_v + c_w \left( T - 273.16 \right)} \right)
\]

+ 10 SGCOL \left( 1 - \frac{c_l \left( T - 273.16 \right)}{L_v + c_w \left( T - 273.16 \right)} \right)

(D.34)

where \( F_s \) is the ventilation factor for graepe, i.e.;

\[
F_s = 1 + 0.265 \left( \frac{\rho_0 D_k}{v_s} \right)^{0.5}
\]

(D.35)

and \( q_{sw} \) is the vapor saturation mixing ratio over liquid water. The factor of 10 multiplying SGCOL increases the collection efficiency from 0.1 to 1.0 for wet growth.

If GRME + PGCOL - PGWG > 0, i.e., more water is collected than can freeze, wet growth is assumed and the excess is shed via precipitation.

\[
SGCOL = 10 \times SGCOL
\]

(D.36)

to change the snow collection efficiency to 1.0. The precipitation water shed is

\[
GPSHD = GRME + PGCOL - PGWG + SGCOL
\]

(D.37)

VSDEP - vapor deposition on snow

This function is from Stephens (1979). The crystals are assumed hexagonal plates. Once they are large enough, mass riming becomes important and the heat release is included in the vapor deposition rate.

\[
VSDEP = \frac{4 D_s SS N_{TG} G(T,p) F_s}{\rho_0} - \frac{L_v^2 G(T,p) \rho_0}{R_T K_T T^2}
\]

(D.38)

where SS is the vapor supersaturation mixing ratio with respect to ice;

\[
SS = \left[ \frac{q_v}{q_{sw}} - 1 \right]
\]

(D.39)

and

\[
G(T,0) = \left[ \frac{L_v^2}{K_T R_T T^2} + \frac{R_T T}{\psi e(T)} \right]^{-1}
\]

(D.40)

\( F_s \) is the ventilation factor for snow.
\[ F_s = 1 + 0.229 \left( \frac{D_s D_t}{\nu_s} \right) \varepsilon ^{0.5} \]  

(D.6)

VGDEP - vapor deposition on graupel

\[ \text{VGDEP} = 2 \pi \text{SS} \text{N}_{10} D_4 G_4(T, p) F_4 \frac{1}{\rho_o} \]  

(D.4)

where the supersaturation SS is defined in equation D.39, and the ventilation factor \( F_4 \) is given in equation D.35.

D.2.2 \( T \geq 273.16 \)

SINIT, SRME, CSFZ, PGFZ, SGCOL, SGCON, VSDEP and VGDEP are set to 0.

The remaining terms are calculated in the following manner;

SMELT - melting of snow

All snow is assumed to melt instantly;

\[ \text{SMELT} = \frac{\Delta h}{\Delta t} \]  

(D.48)

GRME - riming of graupel

Rimming of graupel is calculated using equations D.29 and D.30.

PGCOL - graupel collection of precipitation

This term is calculated using equation D.20.

GMELT - melting of graupel

Melting of graupel is computed using the expression derived by Stephens (1978):

\[ \text{GMELT} = \frac{1}{L_f} \left( \frac{\text{K}_T \left( T - 273.16 \right)}{\rho_o} + 2 \pi D_4 N_{10} F_4 L_o \psi (q_o - q_{iw}) \right) \]  

(D.44)

\[ \left( \frac{\text{GRME + PGCOL}}{T - 273.16} \right) \]

The first term on the right-hand side is the heat conducted from the surrounding air and from water vapor condensing on the particle with the particle temperature set to 273.16 K. The second term is the heat transfer to the graupel from liquid water accreted onto the particle from riming of cloud droplets and collection of precipitation drops.

GPSHD - precipitation shedding due to wet growth of graupel

All water collected by graupel is assumed to shed, i.e.;

\[ \text{GPSHD} = \text{GRME + PGCOL} \]  

(D.45)

D.3 Computation of \( \psi, \text{K}_T \) and \( \mu \)

The molecular diffusivity, \( \psi \), and the dynamic viscosity, \( \mu \), are computed using the expressions from Cheng (1981):

\[ \psi = 2.26 \times 10^{-8} \left( \frac{10000}{T} \right)^{1.81} \]  

(D.46)

and

\[ \mu = 2.616 \times 10^{-5} \left( \frac{T}{296.16} \right)^{1.50} \]  

(D.47)

The thermal conductivity, \( \text{K}_T \) is calculated using the expression (Rogers, 1979)

\[ \text{K}_T = A \mu \]  

(D.48)

where the constant \( A \) has been set here to yield the value of \( \text{K}_T \) at \( T = 273.16 \) found in the CRC Handbook, vol. 52 (1972).

\[ \text{K}_T = 1323.18 \mu \]  

(D.49)

This expression is within 5% of the CRC values for 233 K \( \leq T \leq 293 \) K.
APPENDIX E

Supplemental Figures

I think it important that the interested reader have the opportunity to examine the model output in some detail. However, including all the potentially relevant figures in the main text of this dissertation would have been too disruptive for most readers, therefore this appendix contains incidental figures upon which much of the discussion in chapters 3 through 6 was based.

The figures E.1.1 through E.1.23 are from Chapter 3, figures E.2.1 through E.2.7 are relevant to the discussion in Chapter 4, and figures E.3.1 through E.3.5 are used in Chapter 6.

E.1.1 Contour plot of the entrainment velocity in the inner cloud region for the July 19 base run. Contour interval is 3 m$^{-2}$/s; entrainment is shown as dotted contours, detrainment as solid contours.

E.1.2 Contour plot of the turbulent kinetic energy in the inner cloud region. Contours are from 0 to 36 J/kg; the contour interval is 3 J/kg.
E.1.3 Contour plot of the cloud droplet mixing ratio for the inner cloud region. Contours are from 0.01 to 3.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.

E.1.4 Contour plot of the precipitation mixing ratio for the inner region. Contours are from 0.01 to 3.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.

E.1.5 Contour plot of the snow mixing ratio for the inner region. Contours are from 0.01 to 3.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.

E.1.6 Contour plot of the graupel mixing ratio for the inner region. Contours are from 0.01 to 3.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.
E.1.7 Contour plot of the entrainment velocity in the outer cloud region for the July 19 hour run. Contour interval is 5 m s⁻¹; entrainment is shown as dotted contours, detrainment as solid contours.

E.1.8 Contour plot of the turbulent kinetic energy in the outer cloud region. Contours are from 0 to 80 J/kg; the contour interval is 8 J/kg.

E.1.9 Contour plot of the cloud droplet mixing ratio for the outer cloud region. Contours are from 0.01 to 3.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.

E.1.10 Contour plot of the precipitation mixing ratio for the outer region. Contours are from 0.01 to 3.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.
Contour plot of the snow mixing ratio for the outer region. Contours are from 0.01 to 5.0 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.

Contour plot of the entrainment velocity in the inner cloud region for the August 6 base run. Contour interval is 3 ms⁻¹; entrainment is for dotted contours, detrainment for solid contours.

Contour plot of the graupel mixing ratio for the outer region. Contours are from 0.01 to 5.01 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.
E.1.14 Contour plot of the cloud droplet mixing ratio for the inner cloud region. Contours are from 0.01 to 3.04 g/kg at 0.2 g/kg intervals. The labels are scaled by 100.

E.1.15 Contour plot of the precipitation mixing ratio for the inner region. Contours and scaling as in figure E.1.14.

E.1.16 Contour plot of the snow mixing ratio for the inner region. Contours and scaling as in figure E.1.14.

E.1.17 Contour plot of the graupel mixing ratio for the inner region. Contours and scaling as in figure E.1.14.
E.1.18 Contour plot of the entrainment velocity in the outer cloud region for the August 8 run. Contour interval is 3 m/s²; entrainment is shown as dotted contours, detrainment as solid contours.

E.1.19 Contour plot of the turbulent kinetic energy in the outer cloud region. Contours are from 0 to 10 J/kg; the contour interval is 1 J/kg.

E.1.20 Contour plot of the cloud droplet mixing ratio for the outer cloud region. Contours and scaling as in figure E.1.14.
E.1.21 Contour plot of the precipitation mixing ratio for the outer region. Contours and scaling as in figure E.1.14.

E.1.22 Contour plot of the snow mixing ratio for the outer region. Contours and scaling as in figure E.1.14.

E.1.23 Contour plot of the graupel mixing ratio for the outer region. Contours and scaling as in figure E.1.14.
E.2.1 Inner region vertical velocity contour plots for a variation in \( w_{\text{mult}} \). Contour intervals are 3 m\(^2\) s\(^{-1}\); negative velocities are shown as dashed lines. 4.1a: \( w_{\text{mult}} = 0.5 \), 4.1b: \( w_{\text{mult}} = 1.5 \), 4.1c: \( w_{\text{mult}} = 2.5 \).

E.2.2 Inner region vertical velocity contour plot for the Asai and Kasahara (1997) entrainment formulation. Contour interval is 2 m\(^2\) s\(^{-1}\).

E.2.3 Inner region total liquid water and ice contour plot for the Asai and Kasahara (1997) entrainment formulation. Contours from 0.01 to 3.01 g/kg are plotted; the interval is 0.2 g/kg.
E.2.4 Outer region vertical velocity contour plot for the Assal and Kasahara (1987) entrainment formulation. Contour interval is 2 m s\(^{-1}\).

E.2.5 Outer region total liquid water and ice contour plot for the Assal and Kasahara (1987) entrainment formulation. Contours and interval as in E.2.3.

E.2.6 Inner region vertical velocity contour plot for the Cotton (1975) entrainment formulation. Contour interval is 3 m s\(^{-1}\).

E.2.7 Inner region total liquid water and ice contour plot for the Cotton (1975) entrainment formulation. Contours and interval as in E.2.3.
E.2.14 Outer region vertical velocity contour plot for the TKE entrainment formulation with avl=800 m. Contour interval is 3 m/s

E.2.17 Outer region total liquid water and ice contour plot for the TKE entrainment formulation with avl=800 m. Contours and interval as in E.2.3.

E.2.18 Inner region vertical velocity contour plot for the TKE entrainment formulation with avl=1000 m. Contour interval is 3 m/s.

E.2.19 Inner region total liquid water and ice contour plot for the TKE entrainment formulation with avl=1000 m. Contours and interval as in E.2.3.
E.2.23 Inner region contour plots for $\beta=0.5$. Other parameters are set to the default values listed in table 4.1. (a) the cloud droplet field, (b) the snow field, (c) the graupel field.
Contours and interval as in E.2.3.

E.2.24 Inner region contour plots for $\beta=0.6$. Other parameters are set to the default values listed in table 4.1. (a) the cloud droplet field, (b) the snow field, (c) the graupel field.
Contours and interval as in E.2.3.
E.3.1 Inner cloud region contour plot of NH$_4^+$ aerosol for the Continental Background initialization. Contours are from 0.01 to 1.01 µg/kg; contour interval is 0.1 µg/kg. Note that the contour labels are scaled by 100.

E.3.2 Inner cloud region contour plot of NH$_4^+$ in cloud droplets for the Continental Background initialization. Contours and scaling as in E.3.1.
E.3.3 Inner cloud region contour plot of NH$_4^+$ in precipitation for the Continental Background initialization. Contours and scaling as in E.3.1.

E.3.4 Inner cloud region contour plot of NH$_4^+$ in snow for the Continental Background initialization. Contours and scaling as in E.3.1.

E.3.5 Inner cloud region contour plot of NH$_4^+$ in graupel for the Continental Background initialization. Contours and scaling as in E.3.1.
APPENDIX F

Model Code

This appendix contains the entire model code (5400 lines) and so is somewhat compressed. The format is CDC CYBER update program library, in that the common blocks are defined using *comdeck statements and then inserted into the relevant subroutines upon compilation (*call statements). The code is documented, and the subroutines discussed in the main text are listed in the table of contents. All subroutines and functions are preceded by *deck name lines.

The main program, Walken, sets up the environmental arrays and transfers to subroutine Clouds, which computes the cloud fields.

Good luck!
CALL SYMB (NAME, x, y, z, a, b, c)
CALL SYMB (NAME, x, y, z, a, b, c)

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END IF

END IF
A VARIABLE: \( x \)

CALL GRID
CALL GRID
CALL GRID
CALL GRID
CALL GRID
CALL GRID
CALL GRID
CALL GRID
CALL GRID
CALL GRID

END IF

END IF

END IF

IF (SIMULATION AND CONTACT THEN GO TO 37)

IF (SIMULATION AND CONTACT THEN GO TO 37)

IF (SIMULATION AND CONTACT THEN GO TO 37)

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IF (SIMULATION AND CONTACT THEN GO TO 37)

IF (SIMULATION AND CONTACT THEN GO TO 37)

IF (SIMULATION AND CONTACT THEN GO TO 37)
COHINE X
CONTINUE
DO 100 J=1,JU,LJL
DO 99 I=1,IS,LS
100 CONTINUE
CONTINUE
DO 200 I=1,IS,LS
CONTINUE
DO 500 J=1,JU,LJL
CONTINUE
DO 100 I=1,IS,LS
CONTINUE
DO 99 J=1,JU,LJL
CONTINUE
DO 200 I=1,IS,LS
CONTINUE
DO 500 J=1,JU,LJL
CONTINUE
DO 100 I=1,IS,LS
CONTINUE
DO 99 J=1,JU,LJL
CONTINUE
DO 200 I=1,IS,LS
CONTINUE
DO 500 J=1,JU,LJL
CONTINUE
DO 100 I=1,IS,LS
CONTINUE
DO 99 J=1,JU,LJL
CONTINUE
DO 200 I=1,IS,LS
C  INSERT AND OUTER BOUNDARIES
C  THE BODY AND ITS MOVEMENT ARE CALCULATED HERE.
C  THE REGION MOVEMENT, ACS, AND TORINO ARE DEPICTED IN CLASSES.
C  THE BODY'S MOVEMENT IS COMPUTED FROM THE BODY'S MOVEMENT.
C  DO 158 I = 1, N
C  T (I) = T 
C  DO 156 I = 1, N
C  T (I) = T 
C  DO 154 I = 1, N
C  T (I) = T 
C  DO 152 I = 1, N
C  T (I) = T 
C  DO 150 I = 1, N
C  T (I) = T 
C  DO 148 I = 1, N
C  T (I) = T 
C  DO 146 I = 1, N
C  T (I) = T 
C  DO 144 I = 1, N
C  T (I) = T 
C  DO 142 I = 1, N
C  T (I) = T 
C  DO 140 I = 1, N
C  T (I) = T 
C  DO 138 I = 1, N
C  T (I) = T 
C  DO 136 I = 1, N
C  T (I) = T 
C  DO 134 I = 1, N
C  T (I) = T 
C  DO 132 I = 1, N
C  T (I) = T 
C  DO 130 I = 1, N
C  T (I) = T 
C  DO 128 I = 1, N
C  T (I) = T 
C  DO 126 I = 1, N
C  T (I) = T 
C  DO 124 I = 1, N
C  T (I) = T 
C  DO 122 I = 1, N
C  T (I) = T 
C  DO 120 I = 1, N
C  T (I) = T 
C  DO 118 I = 1, N
C  T (I) = T 
C  DO 116 I = 1, N
C  T (I) = T 
C  DO 114 I = 1, N
C  T (I) = T 
C  DO 112 I = 1, N
C  T (I) = T 
C  DO 110 I = 1, N
C  T (I) = T 
C  DO 108 I = 1, N
C  T (I) = T 
C  DO 106 I = 1, N
C  T (I) = T 
C  DO 104 I = 1, N
C  T (I) = T 
C  DO 102 I = 1, N
C  T (I) = T 
C  DO 100 I = 1, N
C  T (I) = T 
C  DO 98 I = 1, N
C  T (I) = T 
C  DO 96 I = 1, N
C  T (I) = T 
C  DO 94 I = 1, N
C  T (I) = T 
C  DO 92 I = 1, N
C  T (I) = T 
C  DO 90 I = 1, N
C  T (I) = T 
C  DO 88 I = 1, N
C  T (I) = T 
C  DO 86 I = 1, N
C  T (I) = T 
C  DO 84 I = 1, N
C  T (I) = T 
C  DO 82 I = 1, N
C  T (I) = T 
C  DO 80 I = 1, N
C  T (I) = T 
C  DO 78 I = 1, N
C  T (I) = T 
C  DO 76 I = 1, N
C  T (I) = T 
C  DO 74 I = 1, N
C  T (I) = T 
C  DO 72 I = 1, N
C  T (I) = T 
C  DO 70 I = 1, N
C  T (I) = T 
C  DO 68 I = 1, N
C  T (I) = T 
C  DO 66 I = 1, N
C  T (I) = T 
C  DO 64 I = 1, N
C  T (I) = T 
C  DO 62 I = 1, N
C  T (I) = T 
C  DO 60 I = 1, N
C  T (I) = T 
C  DO 58 I = 1, N
C  T (I) = T 
C  DO 56 I = 1, N
C  T (I) = T 
C  DO 54 I = 1, N
C  T (I) = T 
C  DO 52 I = 1, N
C  T (I) = T 
C  DO 50 I = 1, N
C  T (I) = T 
C  DO 48 I = 1, N
C  T (I) = T 
C  DO 46 I = 1, N
C  T (I) = T 
C  DO 44 I = 1, N
C  T (I) = T 
C  DO 42 I = 1, N
C  T (I) = T 
C  DO 40 I = 1, N
C  T (I) = T 
C  DO 38 I = 1, N
C  T (I) = T 
C  DO 36 I = 1, N
C  T (I) = T 
C  DO 34 I = 1, N
C  T (I) = T 
C  DO 32 I = 1, N
C  T (I) = T 
C  DO 30 I = 1, N
C  T (I) = T 
C  DO 28 I = 1, N
C  T (I) = T 
C  DO 26 I = 1, N
C  T (I) = T 
C  DO 24 I = 1, N
C  T (I) = T 
C  DO 22 I = 1, N
C  T (I) = T 
C  DO 20 I = 1, N
C  T (I) = T 
C  DO 18 I = 1, N
C  T (I) = T 
C  DO 16 I = 1, N
C  T (I) = T 
C  DO 14 I = 1, N
C  T (I) = T 
C  DO 12 I = 1, N
C  T (I) = T 
C  DO 10 I = 1, N
C  T (I) = T 
C  DO 8 I = 1, N
C  T (I) = T 
C  DO 6 I = 1, N
C  T (I) = T 
C  DO 4 I = 1, N
C  T (I) = T 
C  DO 2 I = 1, N
C  T (I) = T 
C  DO I = 1, N
C  T (I) = T 
C  STOP
DO 100 I=1,N
  A(J)=(A(J)+ALPHA)
100 CONTINUE

DO 110 J=1,N
  B(J)=(B(J)+ALPHA)
110 CONTINUE

DO 120 I=1,N
  A(I)=(A(I)+B(J))*10.0
  B(I)=(B(I)+B(J))*10.0
120 CONTINUE

DO 130 J=1,N
  A(J)=(A(J)+B(J))*10.0
130 CONTINUE

DO 140 I=1,N
  B(I)=(A(I)+B(I))*10.0
140 CONTINUE

DO 150 J=1,N
  A(J)=(A(J)+B(I))*10.0
150 CONTINUE

END

C CALCULATE THE LOWER BOUNDARY U AND P.
C
C \phi(U) = 0
C
C IF (U=0) THEN
C U = U + 0.001
C CEND IF
C
C IF (P=0) THEN
C P = 0.001
C CEND IF

C CALL GIZM
C CALL GLIO
C CALL GIZM
C CALL GIZM
C
C END
DO 100 I=1,NL
   100   CONTINUE
      DO 99 I=1,NL
         99   CONTINUE
         C EVALUATE THE LOWER BOUNDARY B AND F.
         PLUG IN
         IL=10
         FLAG=88
         END IF
         NO=K(XI)-1+NO+1
         IF (NO.GT.NP) NO=NO-NP+1
         CONTINUE
         C NO=K(XI)-1+NO+1 TO PREVENT
         C A SINGLE ATTAINMENT. C
         C MULTIPLE RECIPROCAL
         C WITH THE SOLUTION.
         EVENT THEN
         IER=10000
         IF (IFLAG.EQ.0) THEN
            DO 100 I=1,NL
               100   CONTINUE
                  C ENTER-sector with the boundary condition flags for left
                  C FLAG for lower B.C. 1
                  C F=10
                  C FLAG for upper B.C. 1
                  C F=10
                  C IF B<0 THEN
                  C RETURN
                  END IF
         END IF
         1)
         C IF B<0 THEN
         C RETURN
         END IF
         150 CONTINUE
         160 CONTINUE
         170 CONTINUE
         180 CONTINUE
DO 170 I=1,IN
   180   CONTINUE
      END
      END
C  GET_Surface from cloud

    IF(Probability cloud > 0.5) THEN
        Probability cloud = 0.5
    END IF

C  RETURN

C  ENTRY

C  FORALL(I,J) WHEN (I,J) NOT IN (1,1) THEN
        EQ(1) = EQ(I,J)
END FORALL

C  RETURN

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END

C  END
C EXPANSION OF POISSON.
C A FUNCTION ARRAY.
C 1 POINT AT WHICH DERIVATIVE IS EVALUATED.
C N STEP SIZE BETWEEN POINTS.
C N+1 ARRAY SIZE.
C 4 FIRST DERIVATIVE CALCULATED BY
C CENTERED SECOND ORDER DERIVATIVE.
C USING 5 POINTS.
C CENTERED FOURTH ORDER DERIVATIVE.
C USING 9 POINTS.
C FOURTH ORDER BACKWARD DIFFERENCE USING
C 5 POINTS SECOND ORDER.
C FOURTH ORDER BACKWARD DIFFERENCE USING
C 5 POINTS SECOND ORDER.
C FOURTH ORDER BACKWARD DIFFERENCE USING
C 5 POINTS SECOND ORDER.
C FOURTH ORDER BACKWARD DIFFERENCE USING
C 5 POINTS SECOND ORDER.
C DIMENSION ARRAY.
C GO TO (LOCAL,MAX,MN,NC).
C LOCAL,MAX,MN,NC.
C 10 DIPOL=MA+I*AD+2*AD*AD).
C RETURN.
C 20 DIPOL=MA+I*AD+4*AD*AD+6*AD*AD).
C RETURN.
C 30 DIPOL=MA+I*AD+6*AD*AD+4*AD*AD).
C RETURN.
C 40 DIPOL=MA+I*AD+4*AD*AD+6*AD*AD).
C RETURN.
C 50 DIPOL=MA+I*AD+2*AD*AD).
C RETURN.
C 60 DIPOL=MA+I*AD+4*AD*AD).
C RETURN.
C 70 DIPOL=MA+I*AD).
C RETURN.
C END.
C WRITEму READи SURFACES NAVIGATOR.DO.
C THIS SUBROUTINE WRITES CLUSTERS PARAMETERS.
C IF A BOOLEAN VARIABLE TO TEMPERATURE.
C AND TO CURRENT.
C WRITE mu SELECTED VARIABLES TO TEMPERATURE.
C WRITE mu SELECTED VARIABLES TO TEMPERATURE.
C WRITE mu SELECTED VARIABLES TO TEMPERATURE.
C WRITE mu SELECTED VARIABLES TO TEMPERATURE.
C LOCAL, NUM.
C LOCAL, NUM.
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C LOCAL, NUM.
C LOCAL, NUM.
C LOCAL, NUM.
C LOCAL, NUM.
C LOCAL, NUM.
C LOCAL, NUM.
C LOCAL, NUM.
C LOCAL, NUM.
C LOCAL, NUM.
FUNCTION THERM(T,THS,THS,Q,F,THEDA,FTHOS)
    T2 = T + 10000
    IF T2 > 10000 THEN
        RETURN
    END IF

    THOS = T - 10000
    IF THOS < 0 THEN
        RETURN
    END IF

    THDA = THS - 10000
    IF THDA < 0 THEN
        RETURN
    END IF

    THEDA = T - THOS - THDA
    IF THEDA < 0 THEN
        RETURN
    END IF

    FTHOS = F - 10000
    IF FTHOS < 0 THEN
        RETURN
    END IF

    FTHDA = F - THOS - THDA
    IF FTHDA < 0 THEN
        RETURN
    END IF

    FTHEDA = F - THOS - THDA - THOS - THDA
    IF FTHEDA < 0 THEN
        RETURN
    END IF

    FTHOS = FTHOS + FTHOS
    IF FTHOS < 0 THEN
        RETURN
    END IF

    FTHDA = FTHDA + FTHDA
    IF FTHDA < 0 THEN
        RETURN
    END IF

    FTHEDA = FTHEDA + FTHEDA
    IF FTHEDA < 0 THEN
        RETURN
    END IF

    RETURN
END

FUNCTION OXY(T)
    O = T * 10000
    IF O < 0 THEN
        RETURN
    END IF

    O = O + O
    IF O < 0 THEN
        RETURN
    END IF

    RETURN
END

FUNCTION H2O(T)
    H2O = T * 10000
    IF H2O < 0 THEN
        RETURN
    END IF

    H2O = H2O + H2O
    IF H2O < 0 THEN
        RETURN
    END IF

    RETURN
END

FUNCTION CO2(T)
    CO2 = T * 10000
    IF CO2 < 0 THEN
        RETURN
    END IF

    CO2 = CO2 + CO2
    IF CO2 < 0 THEN
        RETURN
    END IF

    RETURN
END

FUNCTION N2(T)
    N2 = T * 10000
    IF N2 < 0 THEN
        RETURN
    END IF

    N2 = N2 + N2
    IF N2 < 0 THEN
        RETURN
    END IF

    RETURN
END

FUNCTION O3(T)
    O3 = T * 10000
    IF O3 < 0 THEN
        RETURN
    END IF

    O3 = O3 + O3
    IF O3 < 0 THEN
        RETURN
    END IF

    RETURN
END

FUNCTION H2(T)
    H2 = T * 10000
    IF H2 < 0 THEN
        RETURN
    END IF

    H2 = H2 + H2
    IF H2 < 0 THEN
        RETURN
    END IF

    RETURN
END

FUNCTION F(T)
    F = T * 10000
    IF F < 0 THEN
        RETURN
    END IF

    F = F + F
    IF F < 0 THEN
        RETURN
    END IF

    RETURN
END
**C ARGUMENTS:**
- N REAL ARRAY
- XA REAL ARRAY
- XB REAL ARRAY
- XA REAL ARRAY
- XB REAL ARRAY
- XA REAL ARRAY
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